The 24th Southeastern Analysis Meeting & the 23th Shanks Lectures

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Contents

Abstracts

Shanks Lectures

Whitney's interpolation problem and interpolation I and II
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Fix positive integers $m,\ n$. Let $f : E \to R$ be given, with E an arbitrary given subset of R^n. How can we decide whether f extends to a C^m function F on the whole R^n? If F exists, how small can we take its C^m norm? What can we say about the derivatives of F at a given point? Can we take F to depend linearly on f? What if we require only that F agree approximately with f on E? Suppose E is finite. Can we compute an F whose C^m norm is close to smallest possible? How many computer operations does it take? What if we are allowed to delete a few points from E? The first talk states results, the second talk gives some ideas from the proofs. Many of the results are joint work with Bo'az Klartag.

Invited Talks

On Operator Norm Localization Property
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A metric space X is said to have operator norm localization property if there exists $c > 0$ such that for every $r > 0$, there is $R > 0$ for which, if ν is a positive locally finite Borel measure on X, H is a separable infinite dimensional Hilbert space and T is a bounded linear operator acting on $L^2(X, \nu) \otimes H$ with propagation r, then there exists an unit vector $\xi \in L^2(X, \nu) \otimes H$ satisfying the $diameter(Supp(\xi)) \leq R$ and $||T|| \leq c||T\xi||$.

We prove that a finitely generated group Γ which is strongly hyperbolic with respect to a collection of finitely generated subgroups $\{H_1, \cdots, H_n\}$ has operator norm localization property if and only if each $H_i, i = 1, 2, \cdots, n$ has operator norm localization property. Furthermore we prove the following result. Let π be the fundamental group of a connected finite graph of groups with finitely generated vertex groups G_P. If G_P has operator norm localization property for all vertices P then π has operator norm localization property.

Isomorphic Submodules are Rare

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While Beurling's Theorem implies that each nonzero submodule of the Hardy module on the disk D is isometrically isomorphic to the Hardy module itself, a result of Richter states that for the Bergman module, the only such submodule is the Bergman module itself.

In this talk, I discuss results on this phenomenon for quasi-free Hilbert modules in the multivariate case for bounded domains in C^k showing that the phenomenon is closely related to Hardy-like modules.

Among results discussed are: (1) If the dimension of the quotient is finite, then k = 1 and only the Hardy module is possible if the domain is D. (2) If the module is essentially reductive and has an isomorphic submodule, then it is subnormal.

Positive scalar curvature and non commutative geometry

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Gromov conjectured that "A uniformly contractible complete Riemannian manifold can not have uniformly positive scalar curvature." Scalar curvature is a local invariant of a smooth manifold. This conjecture gives the information about global behavior (contractibility) of the manifold from the local invariant (scalar curvature) of the manifold. In a joint work with G. Yu, we proved that the Gromov conjecture holds for manifolds with subexponential volume growth. In this talk, we will present this result with explanation of why non commutative geometry is involved.

The Small Ball Inequality in all Dimensions

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The Small Ball Inequality concerns a lower bound on the L^∞ norm of sums of Haar functions adapted to rectangles of a fixed volume. The relevant conjecture is improvement of the average case lower bound by an amount that is the square-root log of the volume of the rectangles. We obtain the first non-trivial improvement over the average case bound in dimensions four and higher. The conjecture is known in dimension 2, a result due to Wolfgang Schmidt and Michel Talagrand, with important contributions from Halasz and Temlyakov. Jozef Beck established a prior result in three dimensions, which argument we extend and simplify.

This question is related to (1) Irregularities of Distribution, (2) Probability and (3) Approximation Theory.

Matrices and Varieties

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For any pair $T = (T_1, T_2)$ of commuting matrices, normalized so that both have norm one, there are many polynomials $p(z_1, z_2)$ that annihilate the pair. There is a special choice with the property that the set $V = \{(z_1, z_2) : |z_1| \leq 1, |z_2| \leq 1,\ p(z_1, z_2) = 0\}$ is a spectral set for T, i.e. for any other polynomial q the inequality

$$\|q(T_1, T_2)\| \leq \|q\|_V$$

holds.

I shall discuss how these bordered varieties V arise, and, more generally, some connections between the geometry of varieties and properties of function algebras.

Approximation by analytic matrix functions in L^p

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I am going to speak about recent results obtained jointly with L. Baratchart and F. Nazarov on approximation in L^p of matrix functions on the unit circle by analytic matrix functions. We have obtained quite unexpected results that are quite different from the results in the case of approximation in L^∞.

Some remarks on the dilation theory for commuting d-contractions

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A family in the sense of J. Agler's model theory is a collection of Hilbert space operators which is uniformly bounded in operator norm and closed under the formation of direct sums, restrictions to invariant subspaces, and unital $*$-representations. An extremal in a family is an operator that can only be extended to another operator in the family by taking direct sums. It is Agler's theorem that every operator in a family can be extended to an extremal for that family. Based on this, one can give quick proofs of basic extension and dilation theorems. For example every isometric operator has a unitary extension, every contraction has a co-isometric extension, etc. In this talk I will discuss some applications of the model theory in the multi-variable context.

The Dixmier trace of Bergman space Hankel operators

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If H_f is a (big) Hankel operator on the Bergman space of the disk with a smooth symbol f then we have the following formula for the Dixmier trace of its modulus:

$$Tr_\omega(|H_f|) = c \int_{\partial\mathbb{D}} |\overline{\partial} f \ | d\theta.$$

I will describe some related results, say a bit about the proof of this result and about the range of f for which it holds. I will also describe some extensions and variations of the result.

Singular integrals and perturbation theory

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The theory of singular integral operators, in particular, a theorem by Arocena Cotlar Sadosky about two weight estimates of Hilbert transform is applied to the investigation of delicate spectral properties in perturbation theory of self-adjoint operators. As an application new result about the absence of the embedded singular spectrum for rank one perturbations is obtained.

The talk is based on a joint work with C. Liaw.

The Cauchy Integral, Its Friends and Family: Analysis, Geometry and Combinatorics

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In 1733 count de Buffon asked the question: What is the probability for a needle of length $L < 1$ to intersect a grid of parallel lines on the plane having distance 1 between each other? In 1898 Paul Painlevé asked another question: How can one describe geometrically the compact sets on the plane such that the only functions analytic and bounded in the complement of these sets are constants? At the end of 20th century it became clear that these two questions are closely related. Moreover, they are closely related to a wide variety of problems, from percolation on graphs to electrostatics.

The key words here are “Calderón-Zygmund capacities”.

Capacities with positive kernels, even their non-linear counterparts, are well understood. But, recently the focus has been on capacities with signed, complex or vector-valued kernels. It is usually quite difficult to prove that they are even ”capacities”. In particular, this was the essence of Vitushkin’s question and Tolsa’s answer about one of them, analytic capacity assigned to the Cauchy kernel on the complex plane. Tolsa’s proof does not work in three dimensions, however the corresponding capacity does exist in three dimensions. It is related to the gradient of the fundamental solution for the Lapalace equation, this is an exact counterpart of Cauchy kernel on the plane. We explain the universal approach to proving subadditivity of such new capacities.

We will mention also a pretty amazing connection between Calderón-Zygmund capacities and the usual (but non-linear) capacities.

Already Vitushkin explored a very enigmatic connection between analytic capacity and Geometric Measure Theory. He put forward the question about the "equivalence" of analytic capacity and the so-called Buffon needle probability. We will describe the status of this problem today. This will bring us naturally to a seemingly elementary problem of estimating the Buffon needle probability of a finite collection of disks located in a Cantor pattern. We will indicate the relation of this problem with percolation on graphs, tiling, the Besicovitch projection theorem, and the Kakeya problem.

Boundedness and Compactness of Hankel Operators on the Sphere

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Let S be the unit sphere in $\mathbf{C}^n$ and let $d\sigma$ be the spherical measure on S. Recall that the Hankel operator H_f is defined by the formula $H_f = (1-P)M_f|H^2(S)$, where $H^2(S)$ is the Hardy space on S and $P : L^2(S, d\sigma) \to H^2(S)$ is the orthogonal projection. We show that a large amount of information about the function $f - Pf$ can be recovered from the properties of the Hankel operator H_f. For example, if H_f is compact, then the function $f - Pf$ is necessarily in VMO.

Contributed Talks

On summability of eigenfunction expansions of piecewise smooth functions

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We consider two forms of eigenfunction expansions associated with an arbitrary elliptic differential operator with constant coefficients and order m, that is the multiple Fourier series and integrals. For the multiple Fourier integrals we prove the convergence of the Riesz means of order $s > (N-3)/2$ of piecewise smooth functions of $N \geq 2$ variables. The same result is proved in the case of the $N \geq 3$ dimensional multiple Fourier series.

We prove the following theorems:

Theorem 1. Let $N \geq 3$ and the set Λ be convex. Then for every piecewise smooth function f with the surface of discontinuity Γ the Riesz means $E^s_\lambda f(x)$ of order $s = (N-3)/2$ are uniformly bounded on each compact set $K \subset R^N \setminus \Gamma$. If $s > (N-3)/2$ then

$$\lim_{\lambda \to +\infty} (2\pi)^{-N} \int\limits_{A(\xi)<\lambda} \left(1 - \frac{A(\xi)}{\lambda}\right)^s \hat{f}(\xi) e^{ix\xi} d\xi = f(x)$$

uniformly on $x \in K \subset R^N \setminus \Gamma$.

Theorem 2. (i) Let $N = 2$ and $A(D)$ be an arbitrary homogeneous elliptic differential operator. Then for any piecewise smooth function f with the surface of discontinuity Γ the Riesz means $E^s_\lambda f(x)$ of order $s > -1/2$, defined as in (1), uniformly converge on $x \in K \subset R^2\Gamma$.

(ii) If $s = -1/2$ then the corresponding Riesz means are uniformly bounded on each compact set $K \subset R^2 \setminus \Gamma$.

De Branges-Rovnyak functional-model reproducing kernel spaces: multivariable generalizations

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It is well known that a holomorphic function S on the unit disk with values equal to contraction operators between two Hilbert spaces $\mathcal{U}$ and $\mathcal{Y}$ (i.e., a Schur-class function S) can be realized in the form of the characteristic function $S(z) = D + zC(I - zA)^{-1}B$ of a unitary colligation

$$U = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

(so U is a unitary from $\mathcal{H}\oplus\mathcal{U}$ to $\mathcal{H}\oplus\mathcal{Y}$ for an appropriate Hilbert space $\mathcal{H}$). One convenient way for producing such a realization A, B, C, D is through the associated de Branges-Rovnyak functional model space $\mathcal{H}(S)$ (together with a closely related extended version $\mathcal{D}(S)$) with reproducing kernel function K_S given by $K_S(z, w) = [I-S(z)S(w)^*]/(1-z\overline{w})$. The de Branges-Rovnyak model space can also be used to give a functional-model for the Sz.-Nagy dilation of a contraction operator and for the associated Lax-Phillips discrete-time scattering system. Recently there has appeared a variety of work generalizing various aspects of these ideas to multivariable settings (e.g., where the unit disk is replaced by (1) the unit ball in $\mathbb{C}^d$ (Drury-Arveson space in place of the Hardy space), (2) d-tuples of operators $T_1, \dots, T_d$ on a Hilbert space such that the block row $[T_1 \cdots T_d]$ is a contraction (row-contraction), (3) the unit polydisk in $\mathbb{C}^d$, or (4) d-tuples $T_1, \dots, T_d$ of operators on a Hilbert space such that each T_k is a contraction (the noncommutative polydisk). We focus here on recent work of the speaker (joint with Dmitry Kaliuzhnyi-Verbovetskyi, Cora Sadosky and Victor Vinnikov) on the (commutative) polydisk setting. According to a seminal result of Jim Agler, a holomorphic function S on the polydisk with values equal to operators between two Hilbert spaces $\mathcal{U}$ and $\mathcal{Y}$ can be realized in the form $S(z) = D + C(I - (z_1P_1 + \cdots + z_dP_d)A)^{-1}(z_1P_1+\cdots+z_dP_d)B$ for a unitary colligation U as above mapping $\mathcal{H}\oplus\mathcal{U}$ to $\mathcal{H}\oplus\mathcal{Y}$, where $P_1, \dots, P_d$ is a spanning family of pairwise-orthogonal projection operators on $\mathcal{H}$. We discuss how polydisk versions of de Branges-Rovnyak functional model spaces can be used to provide a concrete functional-model version of this realization theorem. We also discuss connections with Lax-Phillips scattering and uniqueness issues which are here much more intricate than in the single-variable case.

Extremal problems for harmonic gradients

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We consider an extremal problem on the sphere or the hyperplane in R^n, where the BMO distance to a harmonic gradient of a vector field whose tangential component is a gradient is to be minimized. We obtain a Nehari-type theorem where this distance is proved equivalent to the norm of a Hankel-type operator constructed from the additive decomposition of H^1 functions into sums of curl.grad products, as follows from compensated compactness. This result may be used to regularize overdetermined Dirichlet-Neumann issues.

On the ergodic limit of the spectral shift function

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We study Schrödinger operators on $[0, \infty)$ (with the Dirichlet boundary condition at zero) using box approximation. We establish a connection between the characteristics of the absolutely continuous spectrum of the half-line problem and the discrete spectrum of the finite interval problem. More precisely, we prove that

$$\lim_{R\to\infty} \frac{\pi}{R} \int_0^R \big(N_0^r(\lambda) - N^r(\lambda)\big) dr = \delta\big(\sqrt{\lambda}\big),$$

where N_0^r is the counting function for the free Schrödinger operator on the interval $[0, r]$, N^r is the counting function for the operator with potential, and δ is the phase shift associated with the half-line problem.

A Limiting Case for Riesz s-Energies

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Let A be a compact subset of R^p with Hausdorff dimension d. For $0 < s < d$, let $I_s(\mu)$ denote the double integral over $|x-y|^{-s}$ with respect to μ. It is known that there is a unique *equilibrium measure*, μ_s, that minimizes I_s over the set $\mathcal{M}(A)$ of Borel probability measures supported on A. For $s \geq d$, the quantity I_s is not finite for any measure μ in $\mathcal{M}(A)$. We show that, for a class of sets, which includes compact C^1-manifolds, the normalized d-energy defined as

$$\tilde{I}_d(\mu) := \lim_{s\uparrow d} (d-s) I_s(\mu)$$

exists as an extended real number for any measure in $\mathcal{M}(A)$, and is minimized by normalized Hausdorff measure restricted to A denoted by λ_d. Further, we show that μ_s converges in the weak-star topology to λ_d as s approaches d from below.

Some extremal functions in Fourier analysis.

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We obtain extremal majorants and minorants of exponential type for a class of even functions on $\mathcal{R}$ which includes $\log|x|$ and $|x|^{\alpha}$, where $-1 < \alpha < 1$. We also give periodic versions of these results in which the majorants and minorants are trigonometric polynomials of bounded degree. As applications we obtain optimal estimates for certain Hermitian forms, which include discrete analogues of the one dimensional Hardy-Littlewood-Sobolev inequalities. A further application provides an Erdös-Turán-type inequality that estimates the sup norm of algebraic polynomials on the unit disc in terms of power sums on the roots of the polynomials.

Wold-type decompositions for invariant subspaces of the Bergman space

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Aleman, Richter, and Sundberg proved that if a closed subspace M of the Bergman space A^2 is invariant under the shift, that is, if $zM \subset M$, then M is generated by the orthocomplement of zM within M. This result is viewed as a Bergman space version of Beurling's theorem for invariant subspaces of the Hardy space. In this talk, we present some related work inspired by the aforementioned results.

Monotone Thematic Factorizations of Matrix functions

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We study the so-called thematic factorizations of admissible very badly approximable matrix functions. These factorizations were introduced by V. V. Peller and N. J. Young (1994, J. Funct. Anal. 120, 300-343) for studying superoptimal approximation by bounded analytic matrix functions. As shown by Peller and Young, the thematic indices are not uniquely determined by the function itself. However, it was shown by R. B. Alexeev and V. V. Peller (2001, J. Funct. Anal. 179, 309-332) that the indices of a monotone (non-increasing) thematic factorization of an admissible very badly approximable matrix function are uniquely determined by the matrix function itself. We prove that it is always possible to find a monotone non-decreasing thematic factorization for an admissible very badly approximable matrix function. Our proof is constructive.

Spectral pictures of hyponormal 2-variable weighted shifts

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In joint work with Jasang Yoon, we study the spectral pictures of (jointly) hyponormal 2-variable weighted shifts with commuting subnormal components. By contrast with all known results in the theory of (single and 2-variable) weighted shifts, we show that the Taylor spectrum can be disconnected. We do this by obtaining a simple sufficient condition that guarantees disconnectedness, based on the norms of the horizontal slices of the shift. We also show that for every $k \geq 1$ there exists a k-hyponormal 2-variable weighted shift whose horizontal and vertical slices have 1- or 2-atomic Berger measures, and whose Taylor spectrum is disconnected.

We use tools and techniques from multivariable operator theory, from our previous work on the Lifting Problem for Commuting Subnormals, and from the groupoid machinery developed by the speaker and P. Muhly to analyze the structure of C*-algebras generated by multiplication operators on Reinhardt domains. As a by-product, we show that, for 2-variable weighted shifts, the Taylor essential spectrum is not necessarily the boundary of the Taylor spectrum.

Equivalent conditions on direct path of wavelet sets

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A wavelet set E is a measurable set such that its characteristic function modular $\sqrt{2\pi}$ is the Fourier Transform of an orthonormal wavelet. It is an unsolved question that the family of this type characteristic functions is path connected in L^2 norm such that each point in the path, (which is also a wavelet set) is contained in the union of the starting and ending wavelet sets. We present conditions that equivalent to the existence of a direct path.

Transfer function realization for C^*-correspondence valued Schur-Agler classes

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Dritschel and McCullough, using the test function formalism of Jim Agler, have developed an abstract realization theorem for scalar valued functions in Schur Agler classes, along with applications to Agler-Pick interpolation. Muhly and Solel have likewise developed a realization theorem for W^*-correspondence valued functions in what abstractly corresponds to the rather special "disk" case, with a single test function. We discuss recent efforts to unify these two realization theorems.

Valence of Analytic and Harmonic Functions

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Recently we have proposed a notion of Schwarzian derivative for complex-valued harmonic functions, extending the classical definition for analytic functions. Here we discuss criteria for univalence and estimates of valence, derived from bounds on the Schwarzian of a harmonic mapping. Some of the results appear to be new for analytic functions as well.

Cartan type estimates for Riesz transforms

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For given complex numbers $\nu_1, \dots, \nu_N$, and points $y_1, \dots, y_N$, in $\mathbb{R}^d$, $d \geq 2$, we define the set

$$\mathcal{Z}^s(P, \nu) = \{x \in \mathbb{R}^d : |\sum_{j=1}^{N} \frac{x - y_j}{|x - y_j|^{s+1}} \nu_j| > P\}, \quad s > 0, \quad P > 0.$$

Our goal is to give sharp upper bounds for the size of $\mathcal{Z}^s(P, \nu)$. This size will be measured by the Hausdorff content M_h with various gauge functions h. Among other things, we shall characterize all h for which the estimates do not blow up as $N \to \infty$. In this case a routine limiting arguments allow us to extend our bounds to s-Riesz transforms of all finite Borel measures.

The left tangential operator-argument interpolation problem on the noncommutative ball

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We introduce and study a Fock-space noncommutative analogue of reproducing kernel Hilbert spaces (NRKHS) of de Branges-Rovnyaktype.The NRKHS based on the noncommutative Szegö kernel is the unsymmetrized Fock space. The noncommutative Schur multiplier class consists of contractivemultipliers between two such vector-valued Fock spaces. Popescu and others have studied the left tangential operator-argument interpolation problem for such Schur multipliersvia the commutant lifting approach. We discuss how the grassmannian approach to interpolation due to Ball-Helton can be adapted to arrive at a parametrization of the set of all solutions of such a problem. The main tool is an indefinite-metric analogue of recent work of Ball,Bolotnikov and the speaker on Fock Space.

Somewhere Dense Orbits of Tuples of Operators

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In this talk we will discuss when a somewhere dense orbit of a commuting tuple of linear operators is actually dense.

Solution of the $y = x^3$ truncated moment problem

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Let $b = (b_{ij})(i, j \geq 0, i + j \leq 2n)$ denote a real bisequence of degree 2n. In previous work with R.E. Curto, we showed that b has a representing measure supported in a planar algebraic curve of degree one or two, $p(x, y) = 0$, if and only if the associated moment matrix $M(n)(b)$ is positive, recursively generated, has a column dependence relation $p(X, Y) = 0$, and satisfies the "variety condition". In the present work we solve the corresponding problem for measures supported in $y = x^3$, and we show that some essentially new phenomena arise; in particular, the solution is partly algorithmic.

New Classes of Complex Symmetric Operators

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We say that an operator $T \in B(\mathcal{H})$ is *complex symmetric* if there exists a conjugate-linear, isometric involution $C : \mathcal{H} \to \mathcal{H}$ so that $T = CT^*C$. It is known that the class of complex symmetric operators is large, containing all normal operators, the Volterra integration operator, compressed Toeplitz operators (including the compressed shift and Aleksandrov-Clark operators), and operators induced by Hankel and Toeplitz matrices, among others.

In this talk (joint work with W. Wogen), we discuss several recent additions to this family. In particular, we explain why binormal operators, operators which are algebraic of degree two (including all idempotents), and large classes of rank one perturbations of normal operators are complex symmetric. From an abstract viewpoint, these results explain "why" the compressed shift and Volterra integration operator happen to be complex symmetric. Additionally, we describe all complex symmetric partial isometries and highlight some surprises that occur in low-dimensions.

This talk will be accessible to any analyst.

A critical-exponent Balian-Low theorem

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We prove an uncertainty principle for Gabor systems that generalizes the classical Balian-Low theorem. Namely, if f belongs to the Sobolev space $H^{p/2}(\mathbb{R})$ with Fourier transform in $H^{p'/2}$ (with p and p' conjugate exponents), then the Gabor system of modulates and translates of f associated to $\mathbb{Z} \times \mathbb{Z}$ is not a frame for $L^2(\mathbb{R})$.

Gabor Schauder Bases and the Balian–Low Theorem

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The Balian-Low Theorem is a strong form of the uncertainty principle for Gabor systems that form orthonormal or Riesz bases for $L^2(R)$. In this talk we consider the Balian-Low Theorem in the setting of Schauder bases. We prove that weak versions of the Balian-Low Theorem hold for Gabor Schauder bases, but we constructively demonstrate that several variants of the BLT can fail for Gabor Schauder bases that are not Riesz bases. We characterize a class of Gabor Schauder bases in terms of the Zak transform and product A_2 weights.

The relaxed commutant lifting problem as a general H^2 interpolation problem

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We show that the relaxed commutant lifting problem can be seen as a special case of a general interpolation problem for a certain class of operator-valued H^2-functions. In this case the data for this general interpolation problem is a contraction of a particular form that is underlying the relaxed commutant lifting data. It turns out that the two problems are in fact equivalent, as any contraction of the required form appears as the underlying contraction of some relaxed commutant lifting problem. Results on relaxed commutant lifting then provide a representation of all solutions for the general interpolation problem. We consider some applications and questions arising from the equivalence.

Cross Commutators on Backward Shift Invariant Subspaces over the Bidisk

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Let M be an invariant subspace of $H^2 = H^2(\Gamma^2)$ the Hardy space over the bidisk Γ^2 with $M \neq \{0\}$ and $M \neq H^2$. We write $N = H^2 \ominus M$. For $\psi \in L^\infty$, we define the operators R_ψ on M and S_ψ on N by $R_\psi f = P_M(\psi f)$ and $S_\psi f = P_N(\psi f)$, respectively. We talk about the commutativity of $R_{\psi_1(z)}$, $R^*_{\psi_2(w)}$ and $S_{\psi_1(z)}$, $S^*_{\psi_2(w)}$, respectively.

Non-local problems with special gluing conditions for mixed type partial differential equation

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We consider a non-local boundary-value problem with special gluing condition for parabolic-hyperbolic type equation with two lines of type changing. The uniquness of the solution of this problem will be proved by the "abc" method. The existence of the solution will be proved by the method of integral equations. In particular cases under certain conditions to given functions and parameters non-trivial solutions of corresponding homogeneous problem will be found.

Calderon's Problem for Lipschitz Piecewise Smooth Conductivities

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We consider Lipschitz conductivities which are piecewise smooth across polyhedral boundaries in R^3. Using complex geometrical optics solutions for the Schrodinger operators with certain delta function potentials, we obtain global uniqueness for Calderon's inverse conductivity problem.

A Generalization of Beurling's Theorem

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For a Hilbert space K, we define a shift operator S_K on a vector-valued Hardy space $H^2(\Omega, K)$ where Ω is a bounded finitely connected region in the complex plane, whose boundary consists of a finite number of disjoint, analytic, simple closed curves.

We introduce two kinds of quasi-inner functions, and by using quasi-inner functions, we characterize rationally invariant subspaces for the shift operator S_K.

Polynomials defining distinguished varieties

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When studying two variable polynomials and their relation to the two dimensional torus in $\mathbb{C}^2$, one is presented with a number of interesting classes of polynomials, all of which have close ties to rational inner functions and Pick interpolation problems on the bidisk. In this talk, we relate polynomials whose zero set exits the bidisk through the two-torus to a certain class of polynomials with no zeros on the bidisk. The result is a new proof of a representation formula for distinguished varieties with some refinements when the variety has no singularities on the two-torus.

Non-commutative extensions of Wiener's Lemma in frame and time-frequency analysis

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Some recent extensions of Wiener's Tauberian Lemma and their applications to various problems in time-frequency and frame analysis will be presented. The talk includes collaborative results with A. Aldroubi, R. Balan, K. Okoudjou, and T. Strohmer.

Similarity of Operators and Geometry of Eigenvector Bundles

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We characterize the contractive operators that are similar to the backward shift operator on the Hardy class H^2. This characterization is given in terms of the eigenvector bundles of the operators.

Diagonal Toeplitz operators on weighted Bergman spaces

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For any $\alpha > -1$, let $A^2_\alpha(\mathbb{B}_n)$ denote the corresponding weighted Bergman space of the unit ball in $\mathbb{C}^n$. For a bounded measurable function f, let T_f denote the Toeplitz operator with symbol f. In this talk I will discuss some algebraic properties of Toeplitz operators which are diagonal with respect to the standard orthonormal basis of $A^2_\alpha(\mathbb{B}_n)$. In particular, I will show that if $T_{f_1} \cdots T_{f_N} = 0$, where all but possibly one of these operators are diagonal, then one of them must be zero.

On Burbea's mth order Bergman "metric" and Berezin's operator calculus
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I consider the relation between Burbea's mth order Bergman "metric" $B_m(z,v)$ and the classical Bergman metric $B_1(z,v)$ on bounded domains in $\mathbb{C}^n$, and establish that $B_m(z,v)$ is a constant multiple of $B_1(z,v)^m$ on the open unit ball of $\mathbb{C}^n$. As an application to Berezin's operator calculus, I show that functions on the unit ball which are in the range of the Berezin transform satisfy Bloch-type differential inequalities of all orders.

The Horn conjecture for sums of compact self-adjoint operators
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We provide a complete characterization of the possible eigenvalues of compact self-adjoint operators $A, B^{(1)}, B^{(2)}, \ldots, B^{(n)}$ with the property that $A = B^{(1)} + B^{(2)} + \cdots + B^{(n)}$. When all these operators are positive with finite trace, the eigenvalues were known to be subject to certain inequalities which extend Horn's inequalities from the finite dimensional case. Here we find the proper extension of the Horn inequalities to self-adjoint compact operators.

Multipliers, Phases and Connectivity of Wavelets in $L^2(\mathbb{R}^2)$
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Let A be any $d \times d$ real expansive matrix. For any A-dilation wavelet ψ, let $\widehat{\psi}$ be its Fourier transform. A measurable function f is called an A-dilation wavelet multiplier if the inverse Fourier transform of $(f\widehat{\psi})$ is an A-dilation wavelet for any A-dilation wavelet ψ. In this paper, we consider the wavelet multiplier problem in the two dimensional case. We give a complete characterization of all A-dilation wavelet multipliers under the condition that A is a 2×2 matrix with integer entries and $|\det(A)| = 2$. Using this result, we are able to characterize the phases of A-dilation wavelets and prove that the set of A-dilation MRA wavelets is path-connected under the $L^2(\mathbb{R}^2)$ norm topology for any such matrix A.

Limits of the hyperbolic derivative of conformal maps onto smooth domains with corners

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We consider conformal self-maps φ of the unit disk $\mathbb{D}$ such that $\varphi(\mathbb{D})$ is a Jordan domain whose boundary has a corner at a point $\varphi(\zeta)$ of modulus one. We find a relationship between the limit of the modulus of the hyperbolic derivative of φ along certain simple curves in the disk that end at ζ non-tangentially and several angles related with the kind of corner $\partial\varphi(\mathbb{D})$ has at $\varphi(\zeta)$ and with the way we approach ζ.

H^1-BMO duality associated with semigroup of operators on general measure space

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We study H^1 and BMO spaces on general measure spaces and get an analogue of Fefferman's duality theory. Our H^1 space is defined by the analogue of the classical Lusin area integral. A main challenge in our study is to find alternates of classical proofs involving geometric structure of Euclidean spaces.

Bounded Toeplitz Products on the Weighted Bergman Spaces of the Unit Ball

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Let $p > 1$ and let q be a number such that $(1/p) + (1/q) = 1$. We give a necessary condition for the product of Toeplitz operators $T_f T_{\overline{g}}$ to be bounded on the weighted Bergman space of the unit ball A^p_α $(\alpha > -1)$, where both $f \in A^p_\alpha$ and $g \in A^q_\alpha$, as well as a sufficient condition for $T_f T_{\overline{g}}$ to be bounded on A^p_α. Different and simplified techniques are used as compared with the case $p = 2$ when the theorems have been obtained.

The families index theorem without embedding

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The classical Atiyah-Singer index theorem defines the topological index by using an embedding of the (compact) manifold in a Euclidean space. Such an embedding cannot be obtained for more general index theorems, in particular for groupoid index theory. Using ideas of Gennadi Kasparov, Nigel Higson has given a proof of the classical index theorem not using an embedding, but instead using the Connes-Higson asymptotic morphism. The talk will discuss an extension of Higson's approach that applies to the families index theorem, where the index is a K-class.

Aluthge transforms of cyclic operators

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We will discuss under what conditions the Aluthge transform of a cyclic (hypercyclic, supercyclic) operator has the same property.

The spaces of rational functions and a generalized Beurling's theorem

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I will talk my result on the rational version of J. Thomson's theorem. As an application I will speak my recent result on a generalized Buerling's theorem for the shift operator on Hardy spaces for general open subsets in the complex plane.

Nevanlinna-Pick Interpolation for $C + BH^\infty$

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This talk is about the Nevanlinna-Pick problem for the algebras $C + BH^\infty$ where B is an inner function. Our main focus is a Helson-Lowdenslager type invariant subspace theorem for these algebras, a distance formula akin to Nehari's theorem for the classical H^∞ case, and a scalar Nevanlinna-Pick theorem.

Truncated Toeplitz operators

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In this talk we explore truncated Toeplitz operators $A_\phi f := P_\theta(\phi f)$, on the spaces $H^2 \ominus \theta H^2$, where θ is an inner function, $\phi \in L^\infty$, and P_θ is the orthogonal projection of L^2 onto $H^2 \ominus \theta H^2$ (a typical invariant subspace for the backward shift operator). In particular, we focus on the question, first explored by Sarason, when is a bounded linear operator on $H^2 \ominus \theta H^2$ a truncated Toeplitz operator?

Sharp L^2 inequalities for Dirac type operators

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Coauthors: Alexander Balinsky (Cardiff)

The spectrum of the conformal Dirac operator on the n-sphere is used to obtain sharp L^2 inequalities for this operator and several related operators including the conformal Laplacian and Paenitz operator. Stereographic projections are used to obtain similar inequalities in R^n. This spectrum is also used to explain the break down of related sharp inequalities in even dimensions including sharp inequalities for the bi-Laplacian in four dimensions.

Estimates of Oscillatory Integrals

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Consider the operator (1) $Kf(x) = \int_{\mathbb{R}^2} k(x,y)f(y)dy$ with (2) $k(x,y) = \phi(x,y)e^{ig(x,y)}$, g real-valued,with (3) $g(x,y) = x^a \cdot y^b + \Phi_{**}(x^a, y^b)$ and $x, y, a, b \in \mathbb{R}^2$ and $a, b \geq \bar{1}$. Also we suppose for $x, y \geq \bar{1}$ that (4) $|\partial_x^\alpha \partial_y^\beta \Phi_{**}(x,y)| \leq C$, for all α and $\beta \in \mathbb{N}^2$. We prove that if $a_1/b_1 = a_2/b_2$, $b_1 b_2 > 1$ and $a, b \geq \bar{1}$, then $\|Kf\|_p \leq C\|f\|_p$ for $p = 1 + (b_1/a_1)$.

The s-function and the exponential integral

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The order of exponential integrability is an important characteristic of many function spaces. Two prominent examples are the John-Nirenberg and Chang-Wilson-Wolff inequalities

$$\text{(JN)} \quad \langle e^{\varphi - \langle\varphi\rangle_I}\rangle_I \leq A(\|\varphi\|_{BMO}), \qquad \text{(CWW)} \quad \langle e^{\beta(\varphi - \langle\varphi\rangle_I)^2}\rangle_I \leq B(\|S\varphi\|_{L^\infty(I)}),$$

where $\langle\varphi\rangle_I$ is the average of a function φ over an interval I and $S\varphi$ is the square function of φ relative to I. The inequalities are valid for certain ranges of $\|\varphi\|_{BMO}$, $\|S\varphi\|_{L^\infty(I)}$, and β. In recent years, the Bellman function method has efficiently yielded sharp results in both inequalities (including explicit expressions for A and B). In this talk, we prove the new sharp estimate

$$\langle e^{\varphi - \langle\varphi\rangle_I}\rangle_I \leq \langle e^{\frac{1}{2}(S\varphi)^2}\rangle_I$$

and use it to deduce the previously known, as well as new cases of exponential integrability.

Parameter Collapse due to the Zeros in the Inverse Condition
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Helton, Lasserre, and Putinar (2008, Ann. Probability) expose the relationship between three properties of a measure: the conditional triangularity property of the associated orthogonal polynomials, the zeros in the inverse condition of the truncated moment matrix, and conditional independence. I will provide examples of parameter collapse to product structure given that the zeros in the inverse condition holds up to some degree d. Specifically, start with a parameterized family of probability distribution functions; require that the zeros in the inverse condition up to degree d holds; and validate that imposing this restriction on the parameterized family results in a measure with product structure, or at least that conditional independence holds. Algorithms related to parameter collapse are supplied, including the computation of the zeros in the inverse condition up to degree d.

Weighted Composition Operators into Bloch Spaces
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Coauthors: Shûichi Ohno,

We discuss boundedness and compactness of weighted composition operators between reproducing Hilbert spaces of analytic functions on the open unit disk and the Bloch space or the little Bloch space.

Toeplitz Operators and matrices, eigenvalue distribution, spectrum and attracting features
Elizabeth Strouse
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Coauthors: Stefano Serra Capizzano, Deborah Sesana and Benoit Barrusseau

There is a natural way to see a Toeplitz operator as a 'limit' of Toeplitz matrices (by taking the upper right hand corners) and a simple and fun way to see that the average of the eigenvalues of these matrices often converges to the average of the integral of the function on the right set (this is called the 'Szego theorem). I will talk about recent results applying this result to products of Toeplitz operators, and maybe (if there is time) about our attempts to find something out about the spectrum of Bergman space Toeplitz operators.

Extremely non-multiplicative, non-symmetric, non-commutative operator spaces

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Operator spaces with some extreme properties stated in the title will be discussed. Their geometric structure will be described using a model of a shift. These spaces appear prominently in operatro theory, C*-algebras (Cuntz algebras), and E-semigroup theory (continuous tensor product systems).

An integral formula in weighted Bergman spaces

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In order to establish the expansive multiplier property of extremal functions in the Bergman space A^p, Duren, Khavinson, Shapiro and Sundberg made use of an integral formula involving the biharmonic Green function. Using information about the structure of weighted biharmonic Green functions, we discuss the analogous integral formula in the standard weighted Bergman space A^p_α.

Characterizations of Bloch type spaces by divided differences

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Bloch type spaces on the unit disk are characterized by using Newton's divided differences. Several other characterizations of Bloch type spaces involving two or more points as well as applications to polynomial interpolations to Bloch type functions are also given.