ABSTRACTS OF TALKS

The entries in brackets refer to session numbers. See the index on the inside of the back cover, or the short program on the outside of the back cover.

Dynamical Sampling in Shift-invariant Spaces [M-3A]

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We naturally pose the problem of using a dynamical sampling scheme in shift invariant spaces. It turns out that the connection between dynamical sampling in the continuous and the discrete case is not as trivial. We solve a general case, in which the dynamical sampling problem is not reducible to the problem of dynamical sampling in the discrete case.

Breaking the Coherence Barrier in Compressed Sensing Asymptotic Incoherence and Asymptotic Sparsity [M-6B]

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Compressed sensing provides a theory and techniques for the recovery of signals and images from highly incomplete sets of of measurements. The key ingredients that permit this so-called subsampling are (i) sparsity of the signal in a particular basis and (ii) mutual incoherence between such basis and the sampling system. Provided the corresponding coherence parameter is sufficiently small, one can recover a sparse signal using a number of measurements that is, up to a log factor, on the order of the sparsity. Unfortunately, many problems that one encounters in practice are not incoherent. For example, Fourier sampling, the type of sampling encountered in Magnetic Resonance Imaging (MRI), is typically not incoherent with wavelet bases. To overcome this 'coherence barrier' we introduce a new theory of compressed sensing, where sparsity and incoherence are replaced by two new principles: asymptotic sparsity and asymptotic incoherence. When combined with a multi-level random sampling strategy, this allows one to achieve significant subsampling in problems for which standard compressed sensing is limited by the lack of incoherence. Based on this new theory, we draw several important conclusions concerning the use of compressed sensing in problems such as MRI. First, the optimal sampling strategy is contingent on the sparsity structure of the image to be recovered. Second, the success of compressed sensing is highly dependent on the problem resolution.

High Dimensional Function Approximation on Scattered Data [M-11B]

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High dimensional function approximation can arise in a diverse number of circumstances and we specifically focus on the applications of uncertainty quantification and data analysis to demonstrate fast scalable methods that can capture piecewise smooth functions in high dimensions with high order accuracy and low computational burden. Within the field of uncertainty quantification, the developed methods can be used for both approximation and error estimation of stochastic collocation methods where computational sampling can either be guided or come from a legacy database. High dimensional function approximation can also be used in hierarchical data analysis of large, possibly distributed data, to represent statistics of data in a compact form.

Isogeometric Method for the Elliptic Monge-Ampere Equation [M-11A]

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In this talk, we discuss the application of isogeometric analysis to the elliptic Monge-Ampere equation. The elliptic Monge-Ampere equation is a fully nonlinear equation, i.e. nonlinear in the highest order derivatives. The construction of smooth discrete spaces renders isogeometric analysis a natural choice for the discretization for the equation.

Extremal Problems of Discrete Geometry and their Applications to Optimal Recovery Problems [M-3B]

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In this talk we will discuss problems of optimal recovery of operators in various settings and their connections with certain extremal problems of Analysis and Discrete Geometry. Review of known results will be given and new results will be presented. In particular we will present some results about optimal recovery of solutions of certain partial differential equations.

Optimal and Asymptotically Optimal Recovery of Solutions of Elliptic PDE's [M-3B]

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In this talk we will present a method of recovery of a harmonic function based on incomplete information on the boundary function. We will also present the error of recovery that is optimal on a class as well as asymptotically optimal adaptive approximation of the given function.

C^2 Hermite Interpolation by Pythagorean-hodograph Quintic Triarcs [C-13A]

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In this talk, we propose a new method for solving the problem of C^2 Hermite interpolation in plane and space with the help of triarcs composed of Pythagorean Hodograph (PH) quintics. The main idea is to join three arcs of PH quintics at two unknown points – the first curve interpolates given C^2 Hermite data at one side, the third one interpolates the same type of given data at the other side and the second arc is joined to the first and the third arc with C^2 continuity. For any set of C^2 planar boundary data (two points with the associated first and second derivatives) we construct four possible interpolants. Analogously, for the set of C^2 spatial boundary data we find a four-dimensional family of interpolating PH quintic triarcs. We prove that the best possible approximation order in both planar and spatial case is 4. The designed algorithms are presented on several examples.

The Real Beauty of Potential Theory [C-5B]

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Let K be a compact set. The equilibrium measure is a probability measure supported on K minimizing the energy of the system. Its generalization is the balayage measure. We discuss a numerical algorithm of finding the balayage measure. We also show beautiful fractal like images produced by the algorithm.

Nonlinear Approximation in High Dimensions [P-4]

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Nonlinear methods proved to be indispensable in finding relevant approximations in high dimensions. One of the approaches features adaptive partitioning of the domain mirroring the successful techniques used in low dimensions. We extend them to define adaptive piecewise polynomial approximations of a high dimensional function known through a collection of data points distributed according to an unknown probability measure ρ. While the approximation method is defined in full dimension, the quality of fit is evaluated via a norm with respect to the measure ρ that could be, and often is, essentially low dimensional. The paradigm of *sparse occupancy trees* is the main tool to process the data and to find the approximation. It allows scalable algorithms and on-line data assimilation. Two problems from learning theory, binary classification and regression, are considered as examples.

Asymptotic Behavior of Riesz Polarization on Compact Subsets of Smooth Manifolds [M-3B]

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We study the asymptotic behavior of the Riesz polarization problem which is stated in the following way. Let $\omega := \{x_1, \ldots, x_N\}$ be a configuration of N (not necessarily distinct) points on an infinite compact set $A \subset \mathbf{R}^m$. For $s > 0$, we define the quantity

$$M^s(\omega; A) := \min_{x \in A} \sum_{i=1}^{N} \frac{1}{|x - x_i|^s}.$$

Then the N-point Riesz s-polarization constant of A is defined as

$$M_N^s(A) := \max_{\substack{\omega \subset A \\ \#\omega = N}} M^s(\omega; A),$$

where $\#\omega$ denotes the number of elements in the configuration ω (counting the multiplicity). A sequence $\{\omega_N\}_{N=1}^{\infty}$ of configurations on A such that $\omega_N = N$, $N \in \mathbf{N}$, is said to be asymptotically optimal for the N-point s-polarization problem on A if

$$M^s(\omega_N; A) = M_N^s(A)(1 + o(1)), \quad N \to \infty.$$

Let $\mathcal{H}_d$ be the d-dimensional Hausdorff measure in $\mathbf{R}^m$ normalized so that the copy of the d-dimensional unit cube embedded in $\mathbf{R}^m$ has measure 1. Let also β_d be the volume of the d-dimensional unit ball and δ_x be the unit point mass at the point $x \in \mathbf{R}^m$. We obtain the following result.

Theorem. *Let $A \subset \mathbf{R}^m$ be an infinite set representable as a union of finitely many compact sets $A_1, \ldots, A_l$ each of which is contained in a d-dimensional C^1-manifold in $\mathbf{R}^m$, $d \leq m$, and $\mathcal{H}_d(A_i \cap A_j) = 0$, $1 \leq i < j \leq l$. Then*

$$\lim_{N\to\infty} \frac{M_N^d(A)}{N \ln N} = \frac{\beta_d}{\mathcal{H}_d(A)}.$$

Furthermore, if $\mathcal{H}_d(A) > 0$ and $\omega_N = \{x_{i,N}\}_{i=1}^N$, $N \in \mathbf{N}$, is a sequence of asymptotically optimal configurations for the N-point d-polarization problem on A, then in the weak topology of measures we have*

$$\frac{1}{N}\sum_{i=1}^{N} \delta_{x_{i,N}} \xrightarrow{\ *\ } \frac{\mathcal{H}_d(\cdot)|_A}{\mathcal{H}_d(A)}, \quad N \to \infty.$$

This result proves the conjecture made by T. Erdélyi and E.B. Saff. Analogous results for the minimum d-energy on A were earlier obtained by D.P. Hardin and E.B. Saff.

Progress on Hard Thresholding Pursuit [M-6B]

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In this talk, we revisit the recovery of sparse vectors via the hard thresholding pursuit algorithm. Among other new results, we show that the number of iterations necessary for uniform recovery is at most proportional to the sparsity, provided a restricted isometry condition is satisfied. We also consider several variations of the algorithm, including a variation not requiring any prior estimation. In this case, nonuniform recovery in a number of iterations at most proportional to the sparsity is established for Gaussian and random Fourier measurements. The analysis is illustrated by various numerical experiments.

Extending Fundamental Formulas from Classical B-splines to q-Bsplines [C-15A]

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The q-calculus is the calculus of q-derivatives and q-integrals, discrete notions of differentiation and integration. The q-splines are piecewise polynomials whose q derivatives agree at the joins. These q-splines have a canonical basis with compact support, the q-B-splines, analogous to the standard B-splines for classical splines. We present a collection fundamental formulas for q-B-splines analogous to known fundamental formulas for classical B-splines, including recursive algorithms for evaluation and q-differentiation, a divided difference formula for the q-B-splines, a formula for computing divided differences of arbitrary functions by q-integrating certain $1/q$-derivatives of these functions with respect to the q-B-splines, a closed formula for the q-integral of the q-B-splines over their support, a q-analogue of the Marsden identity, a q-analogue of the de Boor-Fix formula, a q-convolution formula for uniform q-B-splines, and a q-analogue of the Hermite-Genocchi formula.

Isogeometric Analysis and T-splines [P-12]

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During this talk, I will give an introduction to isogeometric analysis putting a focus on what are the important features which make splines and NURBS suitable to approximate solutions of partial differential equations in a framework sufficiently general to include engineering oriented applications. Since breaking the tensor product structure is important to gain generality, I will devote part of my talk to explain how local refinement can be allowed in a spline discretizations with special attention to the use of T-splines, and spanning from their approximation properties to the question of optimal assembly of mass and stiffness matrices.

Quadrature with Respect to Refinable Functions on Fixed Nodes [M-16A]

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The issue of integration with respect to a weight function with refinable properties naturally arises in many applications, such as the construction of the discrete counterpart in Isogeometric Analysis. In this talk we present a stable procedure to construct quadrature rules with assigned nodes for integrals involving refinable functions. The process requires in input the refinement mask coefficients and the sequence of nodes only. The corresponding weights are exactly computed by an iterative procedure that does not involve the solution of linear systems. Tests when equispaced points and Chebychev nodes are considered and compared with the behaviour of weighted-Gauss quadrature showing effectiveness of the method.

Phaseless Reconstruction for Fusion Frames [M-3A]

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Phaseless reconstruction for frames was introduced by Balan/Casazza/Edidin and now there is a broad spectrum of results in this area. We will do phaseless reconstruction for fusion frames. In this case, we are given the norms of the projections of a signal onto subspaces of the Hilbert space and want to reconstruct the signal up to phase. We will give a number of results on this problem and its relationship to phaseless reconstruction for frames. Some of these results are quite surprising.

Comparisons of Derivatives of Local Maxima of Lebesgue Functions for Polynomial Interpolation [C-18A]

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In previous papers, we (Chen and Cheney) initiated a new approach to study Lagrange polynomial interpolation. By introducing a parameter $\alpha \in [0, \pi/n]$, we defined a special class of sets of nodes for polynomial interpolation. The zeros of Chebyshev polynomials of the first kind ($\alpha = \pi/(n+1)$) and second kind ($\alpha = \pi/(n+2)$), Chebyshev extrema ($\alpha = \pi/n$), and equidistant nodes ($\alpha = 0$) are particular cases of this class. We also introduced a parameter $s \in [0, 1]$ to indicate the variable′s relative position in each subinterval. By using parameter s, we gave precise formulas of Lebesgue functions for polynomial interpolation at this class of generalized Chebyshev nodes. We also gave a formula of the difference of Lebesgue functions in consecutive subintervals. By using this difference formula, we compared the value of Lebesgue function at each corresponding point in the consecutive subintervals. Therefore, we may compare the local maxima of Lebasgue functions in consecutive subintervals for some α, but not for all α. In particular, the local maxima of Lebesgue functions are strictly decreasing from outside subinterval towards the middle subinterval for $0 \leq \alpha \leq \pi/(n+1)$. To expand the properties of local maxima of Lebesgue functions, in this paper we study the properties of derivatives (with respect to α) of local maxima of Lebesgue functions. We prove that the derivatives of local maxima of Lebesgue functions are strictly increasing from outside subinterval towards the middle subinterval for $0 < \alpha \leq \pi/n$. Therefore, for each pair of consecutive subintervals, there is an $\alpha \in (\pi/(n+1), \pi/n)$, such that the local maxima are equal in this pair of consecutive subintervals for the corresponding Lebesgue function. To obtain these properties, we first give some properties of extrema points, at which the Lebesgue functions obtain their local maxima. However, there is no α such that local maxima are equal for three consecutive subintervals for the corresponding Lebesgue function. In a previous paper we gave a further generalization of this special class of sets of nodes. In addition to the parameter α, each set of nodes also depends on a function with particular properties. We are particularly interested in the functions sin, sin sin, sin sin sin, etc. The derivatives of local maxima of corresponding Lebesgue functions are also strictly increasing from outside subinterval towards the middle subinterval for such a function and $0 < \alpha \leq \pi/n$. There is a function and an α for which, the local maxima are all equal for the corresponding Lebesgue function. We believe that this function can be determined and the value of the parameter can be exactly calculated. (The complete title of this paper is "Comparisons of Derivatives of Local Maxima of Lebesgue Functions for Polynomial Interpolation at the Generalized Chebyshev Nodes".)

Free-form Subdivision Surfaces and the Helfrich Model [M-16B]

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The Helfrich model has been used to explain the shape of biological membranes. To explore this model, we numerically approximate the membrane using subdivision surfaces, a technique commonly used in the computer aided geometric design community. We explain how to accurately compute the Willmore energy and its gradient vector with respect to the control values of a subdivision surface, which enables an accurate computer simulation of the Helfrich model when combined with a nonlinear optimization solver. We also show seemingly new phase transition phenomena in the Helfrich model observed based on our numerical method.

Numerical Methods for High-Dimensional Response-Excitation PDF Equations [M-11B]

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Evolution equations of the joint response-excitation probability density function (REPDF) generalize the existing PDF evolution equations and enable us to compute the PDF of the solution of stochastic systems driven by colored random noise. We study the theoretical and numerical issues arising in this formulation including high dimensionality and discontinuity. Particularly, we adopt various numerical techniques such as adaptive discontinuous Galerkin (DG) method and probabilistic collocation method (PCM) combined with sparse grid, ANOVA decomposition, and proper generalized decomposition (PGD). In addition, we propose small noise approximation and Monte Carlo based conditional averaging. Finally, the proposed numerical methods are demonstrated in Duffing oscillator, tumor cell model, and nonlinear advection equation.

Spatial-spectral Fusion via Data Dependent Operators [M-3A]

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In this talk we present a deterministic algorithm which exploits fused representations of certain well known data-dependent operators, such as, e.g., graph Laplacian or Schroedinger operators. By means of these operators we introduce the notion of fusion and integration of heterogeneous data. We verify our approach by applying it to the problem of spatial-spectral fusion, which is a longstanding problem in the Hyperspectral Satellite Imagery (HSI) community. It deals with finding effective and efficient ways to integrate the spectral analysis methods, while taking advantage of the knowledge about the location.

On Convergence of Singular Integral Operators Depending on Three Parameters with Radial Kernels [C-13B]

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In this paper we will prove the pointwise convergence of $L(f;x,y,\lambda)$ to $f(x_0,y_0)$, as (x,y,λ) tends to $(x_0,y_0,{}_0)$ in the L_2 space, by the three parameter family of singular operators. In contrast to previous work, where x_0 is a μ-generalized Lebesgue point of f, we take the kernel function radial.

GERBS-based Intermediate Approximation on Triangulated Domains [C-13A]

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In the present talk a new type of intermediate approximation of regular distributions (locally integrable functions) for the case of triangulations on two-dimensional domains will be proposed as an alternative to the Steklov-means. This construction relies on a novel convex smooth (possibly infinitely smooth) resolution of unity over triangulated domains, based on generalized expo-rational B-splines (GERBS). The extension of this construction from the bivariate to the n-variate case is relatively straightforward, thanks to the possibility to extend the definition of the afore-mentioned smooth convex partition of unity to dimensions higher than 2. One application of the new construction is to use it to improve the bounds for the constants of equivalence between moduli of smoothness and respective K-functionals on Lipschitz, as well as on non-Lipschitz cuspidal, domains. Besides this type of application, the smooth B-spline-type basis functions forming the resolution of unity, which are supported on the star-1 neighborhoods of the vertices in the triangulation, as well as their rational (NURBS-type) forms, can be used for a wide range of applications in both finite element analysis and geometric modeling of smooth manifolds with possible singularities and, as such, have the potential of becoming a new versatile tool of isogeometric analysis. More details about some of these latter applications will be given in the talk of Peter Zanaty at the conference.

On the Global and Linear Convergence of the Generalized Alternating Direction Method of Multipliers [M-1B]

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The alternating direction method of multipliers (ADMM) is very effective at solving many practical optimization problems and has wide applications in areas such as compressive sensing, signal and image processing, machine learning and statistics. However, its effectiveness has not been matched by a provably fast rate of convergence; only sublinear rates such as $O(1/k)$ and $O(1/k^2)$ were recently established in the literature. In this talk, We propose a generalized ADMM framework that allows the subproblems to be solved faster and less exactly in certain manners. We show that global linear convergence can be achieved under various scenarios. The derived rate of convergence also provides some theoretical guidance for optimizing the ADMM parameters.

Deterministic Guaranteed Automatic Numerical Algorithms for Univariate Approximation [C-13B]

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Function recovery is a fundamental example of numerical problems that arise often in practice. It is desirable to have automatic algorithms for solving these problems, i.e., the algorithm decides adaptively how many and which pieces of function data are needed and then uses those data to construct an approximate solution. The error of this approximation should be guaranteed not to exceed the prescribed tolerance, and the number of data required by the algorithm should be reasonable, given the assumptions made about the problem and the error tolerance required. A deterministic guaranteed automatic algorithm will be given. The reliability of the algorithm is proved and illustrated by numerical example.

Shellability and Freeness of Continuous Splines [M-8B]

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We provide an example of a two dimensional shellable polyhedral complex P such that the module of splines $C^0(\hat{P})$ is not a free module over the polynomial ring in three variables, answering a question raised in [Schenck,*Equivariant Chow Cohomology of Non-Simplicial Toric Varieties*,2012].

Optimal s-Energy Configurations under External Field [M-3B]

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A configuration of points on the sphere that minimizes discrete Riesz s-energy in the presence of an external field Q is called Q-optimal s-configuration. We show that for $d-2 \leq s < d$ and under some general assumptions on Q such configurations are well-separated.

Approximation of Set-valued Functions in the Symmetric Difference Metric [M-16B]

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We consider the approximation of set-valued functions from finite collections of their samples. Differently from previous works on this subject, our methods are developed and analyzed in the metric space of Lebesgue measurable sets endowed with the symmetric difference metric. The development of new approximation methods is based on the construction of two new weighted averages of sets. We consider two approaches; one based on repeated weighted averages of two sets, and the second based on weighted averages of several sets. In the first approach, we construct subdivision schemes refining general subsets of Euclidean spaces and study their applications to the approximation of univariate set-valued functions, and in particular to the "reconstruction" of 3D objects from their 2D cross-sections. In the second approach, we study the approximation of univariate and multivariate set-valued functions, by adapting classical positive sample-based approximation operators, replacing averages of numbers by averages of sets. Several examples illustrating our results are included in the talk. This talk presents joint works with Shay Kels.

Isogeometric Collocation: Cost Comparison with Galerkin Methods and Extension to Hierarchical NURBS Discretizations [M-16A]

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We compare isogeometric collocation with isogeometric Galerkin and standard finite element methods with respect to the cost of forming the stiffness matrix and residual vector, the cost of direct and iterative solvers, accuracy versus degrees of freedom, and accuracy versus computing time. On this basis, we show that isogeometric collocation has the potential to increase the computational efficiency of isogeometric analysis and outperform both isogeometric Galerkin and standard finite element methods. We then explore an adaptive isogeometric collocation method that is based on local hierarchical refinement of NURBS basis functions and collocation points derived from the corresponding multi-level Greville abscissae. We introduce the concept of weighted collocation that can be consistently developed from the weighted residual form and the two-scale relation of B-splines. Using weighted collocation in the transition regions between hierarchical levels, we are able to reliably handle coincident collocation points that naturally occur for multi-level Greville abscissae. The resulting method combines the favorable properties of isogeometric collocation and hierarchical refinement in terms of computational efficiency, local adaptivity, robustness, and straightforward implementation.

Learning Theory Approach to Minimum Error Entropy Criterion [C-18A]

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We consider the minimum error entropy (MEE) criterion and an empirical risk minimization learning algorithm when an approximation of Rényi's entropy (of order 2) by Parzen windowing is minimized. This learning algorithm involves a Parzen windowing scaling parameter. We present a learning theory approach for this MEE algorithm in a regression setting when the scaling parameter is large. Consistency and explicit convergence rates are provided in terms of the approximation ability and capacity of the involved hypothesis space. Novel analysis is carried out for the generalization error associated with Rényi's entropy and a Parzen windowing function, to overcome technical difficulties arising from the essential differences between the classical least squares problems and the MEE setting.

Group-theoretic Constructions of Erasure-robust Frames [M-3A]

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A recently proposed method for phase retrieval requires frames which are robust against the removal of any fixed proportion of the frame elements. However, such numerically erasure robust frames (NERFs) are difficult to construct explicitly since, like RIP matrices, they must satisfy a combinatorially large number of conditions. We discuss a new method for constructing such frames. We begin by focusing on a subtle difference between the definition of a NERF and that of an RIP matrix, one that allows us to introduce a new computational trick for quickly estimating NERF bounds. In short, we estimate these bounds by evaluating the frame analysis operator at every point of an epsilon-net for the unit sphere. We then borrow ideas from the theory of group frames to construct explicit frames and epsilon-nets with such high degrees of symmetry that the requisite number of operator evaluations is greatly reduced. We conclude with numerical results, using these new ideas to quickly produce decent estimates of NERF bounds which would otherwise take an eternity.

Kernel Methods for Image Reconstruction from Photoacoustic Data [M-6A]

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The reconstruction of images from spherical mean data play a central role in photoacoustic tomography-a rapidly developing modality for in vivo imaging. We present a algebraic reconstruction method based on positive definite kernels rep. radial basis functions. We will present error estimates, discuss fast and stable algorithms, and some numerical examples.

Barycentric Coordinates for Polyhedral Finite Elements [P-7]

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Generalizations of barycentric coordinates to polygons and polyhedra, such as Wachspress and mean value coordinates, have found various applications in curve and surface modelling and computer graphics, such as surface parameterization and curve and surface deformation. More recently it has been shown that the polygonal coordinates can be used as nodal shape functions for finite element methods over polygonal partitions of 2D domains. To prove convergence one must derive bounds on the gradients of these functions in terms of the geometry. In this talk we discuss these developments and focus on recent progress on deriving similar bounds for Wachspress/Warren coordinates over convex polyhedra, which is joint work with Gillette and Sukumar.

Computing Dimension Formulas for Multivariate Spline Spaces [M-8B]

Simon Foucart
Drexel University, Philadelphia, PA
Simon Foucart

I will present a method to generate formulas for the dimensions of the spaces $\mathcal{S}_d^r(\Delta_n)$ of $\mathcal{C}^r$ splines of degree $\leq d$ in n variables over a simplicial partition Δ_n. The method relies on the determination of the sequence $(\dim(\mathcal{S}_d^r(\Delta_n)))_{d\geq 0}$ as a whole by way of the Hilbert series $\sum_{d\geq 0} \dim(\mathcal{S}_d^r(\Delta_n))\, z^d$. I will show how this method is implemented in Sage and demonstrate how it is used to obtain dimension formulas for many tetrahedral partitions (Alfeld splits, type-I split, Worsey–Farin split, generic octahedron, generic 8-cell, etc.). The method also extends to non-simplicial partition such as T-meshes. This is joint work with Patrick Clarke and Tatyana Sorokina.

Spectral Methods for Image Reconstruction from Spherical Means [M-6A]

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In various models of photoacoustic tomography, the reconstruction problem amounts to the inversion of the spherical mean Radon transform. For this purpose, many different exact inversion formulas have been developed. However, in practice only an approximate reconstruction can be computed. In this talk, we present two approximate reconstruction techniques. In the first part of the talk, a summability approach will be introduced as method for approximate inversion of the spherical mean Radon transform. In the second part, we investigate the discretization of series expansion methods for the inversion of the spherical mean Radon transform in an approximation theoretic setting. In particular, we will show that by applying spectral methods to discretization of series expansions, optimal convergence rates can be achieved. Moreover, we shall also outline that these discretization schemes may also be applied in situations where the spherical mean data is available on scattered points.

Superconvergent Derivative Approximation with Periodic Kernels [M-6A]

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In this talk we consider the following kernel-based approximation process: given samples of a periodic target function, an approximation of the m^{th} derivative is obtained by m successive applications of the operator "interpolate, then differentiate" - this process is known in the spline community as *successive splines* or *iterated splines*. When using odd-degree splines on a uniform mesh, this iterated method surprisingly approximates all derivatives at the same rate of convergence, provided the error is only measured at the data sites. We will revisit this iterated scheme with a more general class of kernel interpolation methods in mind, and show that several radial basis function kernels restricted to the circle exhibit similar accelerated convergence properties. Finally, we consider possible extensions to higher-dimensional periodic domains with examples on the 2-sphere and a torus.

Interpolation and Cubature Approximations for a Class of Wideband Integrals on the Sphere [M-8A]

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We propose, analyze, and implement interpolatory approximations and Filon-type cubature for efficient and accurate evaluation of a class of wideband generalized Fourier integrals on the sphere. The analysis includes derivation of (i) optimal order Sobolev norm error estimates for an explicit discrete Fourier transform type interpolatory approximation of spherical functions; and (ii) a wavenumber explicit error estimate of the order $\mathcal{O}(\kappa^{-\ell}N^{-r_\ell})$, for $\ell = 0, 1, 2$, where κ is the wavenumber, $2N^2$ is the number of interpolation/cubature points on the sphere and r_ℓ depends on the smoothness of the integrand. Consequently, the cubature is robust for wideband (from very low to very high) frequencies and very efficient for highly-oscillatory integrals because the quality of the high-order approximation (with respect to quadrature points) is further improved as the wavenumber increases. This property is a marked advantage compared to standard cubature that require at least ten points per wavelength per dimension and methods for which asymptotic convergence is known only with respect to the wavenumber subject to stable of computation of quadrature weights. Numerical results in this article demonstrate the optimal order accuracy of the interpolatory approximations and the wideband cubature.

Optimal C^3 Interpolatory Subdivision Schemes with Fractal Limit Curves [C-10A]

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In this talk, I will consider a one-parameter class of interpolatory subdivision schemes, and provide parameter intervals on which subdivision convergence and regularity is guaranteed. It is then shown that, for parameter values inside certain sub-intervals of the convergence intervals, fractal subdivision curves are obtained. Graphical illustration are provided.

Isogeometric Models based on Hierarchical B-spline Constructions [M-11A]

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The hierarchical B–spline approach has been recently considered as an effective adaptive approximation model in the context of isogeometric analysis. Hierarchical B–splines enable adaptive surface modeling by considering different levels of resolution. A selection procedure allows to identify the set of B–splines which are active at a certain hierarchical level. The set of all active B–splines collected together with respect to a specific subdomain hierarchy constitutes a basis for the corresponding multilevel spline space. Unfortunately, this construction does not preserve the partition of unity property. The recently introduced truncated hierarchical B–spline (THB) model provides a solution to this problem and improves the sparsity pattern related to the matrices which characterize the approximation algorithm. The talk will present recent results related to the application of THB–splines in isogeometric analysis.

Basis Functions for Serendipity Finite Element Methods [M-16A]

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The degree r tensor product finite element method on a cubical mesh in d dimensions employs $(r+1)^d$ basis functions per element, resulting in $O(h^r)$ a priori error convergence rates. While it has long been observed that the same rate of convergence can be obtained in practice with fewer than $(r+1)^d$ degrees of freedom, the mathematical foundation for this phenomenon was only recently made precise in work by Arnold and Awanou [*Found. Comput. Math.* 11 (2011) 337–344]. The basis functions we define have a canonical relationship both to the finite element degrees of freedom as well as to the geometry of their graphs; this makes the bases ideal for applications employing *isogeometric analysis* where domain geometry and functions supported on the domain are described by the same basis functions. Applications to computational cardiac electrophysiology and extensions to higher order serendipity methods will also be discussed.

Towards a General Unified Theory of Classical and Quantum B-Splines [C-13A]

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Classical splines are piecewise polynomials whose classical derivatives up to some order agree at the joins. Quantum splines are piecewise polynomials whose quantum derivatives (the h or q derivatives) up to some order agree at the joins. Generalized quantum splines are piecewise polynomials whose finite differences or generalized quantum derivatives up to some order agree at the joins. Just like classical and quantum splines, generalized quantum splines admit a canonical basis with compact support: the generalized quantum B-splines. These generalized quantum B-splines include both the classical and quantum B-splines as special cases and therefore unify the theory of classical and quantum B-splines inside one general framework. Here we study generalized quantum B-spline bases and generalized quantum B-spline curves, using a very general variant of the blossom, the generalized quantum blossom. Given d non constant linear functions L(t)=(L1(t),...,Ld(t)), the corresponding generalized quantum blossom of a polynomial P(t) of degree d is the unique symmetric, multiaffine function p(u1,...,ud;L) that reduces to P(t) along the L(t)-diagonal. Applying the generalized quantum blossom, we develop algorithms and identities for generalized quantum B-spline bases and generalized quantum B-spline curves, including generalized quantum variants of the de Boor algorithms for recursive evaluation and generalized quantum differentiation, knot insertion procedures for converting from generalized quantum B-spline to piecewise generalized quantum Bezier form, and a generalized quantum variant of Marsden's identity.

New Asymptotic Lower Bound for Cardinality of Spherical Designs [M-3B]

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A new asymptotic lower bound for minimal number of points in a spherical design is proved. The bound involves spherical packing constant in the Euclidean space. For high order designs the estimate is better than known estimates of Delsarte – Goethals – Seidel and Yudin.

Viewing Image Registration as an Approximation Problem [C-5A]

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Image registration is the process of finding correspondence between all points in two images of a scene using the correspondence between a small number of points in the images. In this study, the problem of image registration is treated as one of approximation to scattered data. Extensions of well-known approximation methods to elastic image registration are studied. Among the extensions considered are parametric Shepard interpolation and weighted linear approximation, which produce superior accuracy when compared to thin-plate spline in the registration of multiview aerial images.

Intrinsic Localization of Anisotropic Frames [M-1A]

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We study localization properties of anisotropic frame systems such as curvelets and shearlets. Our main result roughly says that, if a curvelet or shearlet frame is localized in the sense that its Gram matrix has strong off-diagonal decay, then a similar property holds for the canonical dual frame. Such knowledge about the properties of dual frames is of importance for several aspects such as operator compression or characterization of function spaces by frame coefficients.

Optimal A Priori Discretization Error Bounds for Geodesic Finite Elements [M-16B]

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In a number of applications one is led to solve a variational problem where the sought solution is constrained to take its values in a Riemannian manifold. Some examples include harmonic maps into manifolds, variational interpolation in manifolds, or Cosserat rod modeling. Recently, geodesic finite elements have been introduced for the numerical discretization of such problems. They have an elegant covariant formulation and satisfy various desirable equivariance properties. We prove optimal bounds for the discretization error of geodesic finite elements for manifold-valued partial differential equations in variational form which satisfy a coercivity property. For this we generalize both the well-known Cea lemma and the Bramble-Hilbert lemma to manifold-valued functions. Both of these results are interesting in their own right. Our theory, which gives a complete extension of well-established linear results, is purely intrinsic and does not rely on embeddings of the codomain into Euclidean space. Special cases are optimal a priori error estimates for the numerical computation of smooth harmonic maps into manifolds. This is joint work with Hanne Hardering (FU Berlin) and Oliver Sander (RWTH Aachen).

Kernel Interpolation and Quadrature with Localized Bases [M-6A]

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In applying kernel-based approaches to scattered data fitting, quadrature and many other computational problems, one frequently encounters challenges in using the standard basis: as the dimensions of the underlying spaces increase, the practicality of working directly with linear combinations of kernels decreases rapidly. A popular and enduring strategy has been to apply a preconditioner, by way of localizing the kernel, to treat such challenges. Prior attempts in this direction have not yielded reliably local or stable bases for large problems. Despite several methodologies for localizing radial basis functions, none presents a scalable solution: as the underlying spaces increase in size, the performance of the preconditioning method deteriorates. In this talk, we present a simple algorithm for constructing localized bases for some kernel spaces generated by quasi-uniformly distributed centers on certain compact Riemannian manifolds. This algorithm is nearly stationary and has complexity $\mathcal{O}(N(\log N)^d)$ where N is the dimension of the underlying space of kernels and d is the dimension of the manifold. Furthermore, it produces a basis that can be shown to be stable and well localized, regardless of the size N. We will discuss interpolation with this basis as well as a new quasi-interpolation scheme that has optimal approximation orders and works directly on scattered data. This idea has recently lead to a kernel-based quadrature scheme having fast rates of convergence and with weights that can be computed rapidly and stably.

n-term Approximation of Elliptic PDEs

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We investigate the Besov regularity for solutions of elliptic PDEs on polyhedral domains. This is based on regularity results in weighted Sobolev spaces (Babuska-Kondratiev spaces) $\mathcal{K}^m_a(D)$. Following the argument of Dahlke and DeVore, we first prove an embedding of these spaces into the scale $B^r_{\tau,\tau}(D)$ of Besov spaces with $\frac{1}{\tau} = \frac{r}{d} + \frac{1}{2}$. The latter scale is known to be closely related to n-term approximation w.r.t. wavelet systems, and recently this has been extended to adaptive Finite Element approximation. This ultimately yields the rate $n^{-r/d}$ for $u \in \mathcal{K}^m_a(D) \cap H^s(D)$ for $r < r^* \leq m$. In order to obtain the full rate $n^{-m/d}$ we subsequently leave the scale $B^r_{\tau,\tau}(D)$ and instead consider the spaces $B^m_{p,\infty}(D)$. We show that under appropriate conditions we now have an embedding $\mathcal{K}^m_a(D) \cap H^s(D) \hookrightarrow B^m_{p,\infty}(D)$ for some $0 < p \leq p_0 \leq 2$, $\frac{1}{p_0} = \frac{m}{d} + \frac{1}{2}$, which in turn yields the desired approximation rate.

Generalized B-Splines with Complex Orders [C-10B]

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We shall present a generalization of B-spline functions with complex order parameters motivated by the interrelationship between the classical Eulerian numbers and classical B-splines. The basic properties of the complex order B-splines including their recursive relation, Fourier transform, convolution, and asymptotic properties are given. The interrelationship between the values of the generalized B-splines and generalized Eulerian functions of complex orders has been established. Some properties specially for the generalized B-splines with real fraction orders are also obtained by using the interrelationship between the generalized B-splines and the generalized Stirling functions with fraction orders.

Barycentric Interpolation [P-19]

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In this talk I will focus on the method of barycentric interpolation, which ties up to the ideas that

August Ferdinand Möbius published in his seminal work *Der barycentrische Calcül* in 1827. For univariate data, this gives a special kind of rational interpolant which is guaranteed to have no poles and favourable approximation properties. I further discuss how to extend this idea to bivariate data, where it leads to the concept of generalized barycentric coordinates and to an efficient method for interpolating data given at the vertices of an arbitrary polygon.

Generalized Lane–Riesenfeld Algorithms [M-16B]

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The Lane–Riesenfeld algorithm for generating uniform B-splines provides a prototype for subdivision algorithms that use a refine and smooth factorization to gain arbitrarily high smoothness through efficient local rules. In this paper we generalize this algorithm by maintaining the key property that the same operator is used to define the refine and each smoothing stage. For the Lane–Riesenfeld algorithm this operator samples a linear polynomial, and therefore the algorithm preserves only linear polynomials in the functional setting, and straight lines in the geometric setting. We present two new families of schemes that extend this set of invariants: one which preserves cubic polynomials, and another which preserves circles. For both generalizations, as for the Lane–Riesenfeld algorithm, a greater number of smoothing stages gives smoother curves, and only local rules are required for an implementation.

Minimal Growth Sparse Grids for Interpolation and Quadrature [M-11B]

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Sparse grids have been used extensively to delay the curse of dimensionality when interpolating and integrating high-dimensional functions. Sparse grids are most efficient when the underlying one-dimensional quadrature rules are nested. However the number of nodes employed by such nested rules typically grows exponentially with the level of the quadrature rule. In this talk we use one at a time nested one-dimensional Leja quadrature rules to build multi-dimensional sparse grids that grow at the smallest possible rate. Convergence will be demonstrated numerically for a number of examples.

On the Asymptotics of Discrete Riesz Energy on Ahlfors-David Regular Sets [C-5B]

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Discrete minimal s-Riesz energy ($s \geq 0$) of a system of n points on a compact set $\Omega \subseteq \mathbf{R}^d$ has been intensively studied during the last years. For the cases, in which its continuous counterpart exists (i.e. $0 \leq s < d$) the normalized discrete minimal s-Riesz energy is known to converge to the continuous s-Riesz energy as n tends to infinity. For the sphere in $\mathbf{R}^d$, explicit error bounds follows from results due to Wagner and due to Kuijlaars and Saff. In this work we generalize these error bounds to the class of *Ahlfors-David d-regular* sets Ω in the case of (super) harmonic potentials, i.e., $0 \leq s \leq d-2$. Since the equilibrium measure is concentrated on the boundary of Ω for (super) harmonic potentials, we obtain the same error estimates for sets bounding a d-regular set in the sense of Ahlfors-David. The proofs are based on a generalization of an argument due to Tsuji, which he employed in the context of harmonic potentials in the complex plain. In addition, our regularity assumption is discussed.

Polynomial Approximation of Hölder Weighted Integrable Functions [C-18B]

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Hölder spaces of periodical real-valued integrable functions are extended to the case of weighted functions. The translation of functions together with the weights is shown to be more convenient for defining weighted increments. Many known properties of Hölder spaces of integrable functions can be easily obtained in this new context by isometry, while new problems appear in scene. A special case of asymmetric norms arises with the use of sign sensitive weights. The main goal of this short communication is to characterize the weights for which trigonometric polynomials are dense in these Hölder spaces. Some brief historical comments are presented.

Multivariate Approximation with Trigonometric Polynomials from Samples along Generated Sets [M-11B]

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The approximation of problems in d spatial dimensions by sparse multivariate trigonometric polynomials supported on known frequency index sets $I \subset \mathbf{Z}^d$ is an important task with a variety of applications. The use of a generalisation of rank-1 lattices as spatial discretisations offers a suitable possibility for sampling functions and compute the approximating trigonometric polynomials. In particular, we can compute the approximation of a function using one one-dimensional nonequispaced fast Fourier transform. We discuss necessary and sufficient conditions on the generated set guaranteeing stability of the corresponding Fourier matrix. This leads to a fast search algorithm for suitable generated sets. In addition, we analyse the error of the approximation depending on the decay of the Fourier coefficients of the approximated function.

Copositive Approximation By Elements of Finite Dimensional Spaces [C-15A]

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If M is a subspace of $C(Q)$, $f \in C(Q)$ and $g \in M$, then g is said to be "copositive" with f on Q if $f(x)g(x) \geq 0$ for all $x \in Q$. The element $g_0 \in M$ is called a "best copositive approximation" for f from M iff g_0 is copositive with f on Q and $\|f - g_0\| = \inf\{\|f - g\| : g \in M, \text{and } g_0 \text{ is copositive with } f \text{ on } Q\}$. If Q is a compact subset of real numbers, then the n-dimensional subspace M of $C(Q)$ is called a "Strong Chebyshev subspace" of $C(Q)$ if each $g \neq 0$ in M has at most $n - 1$ zeros, and at most $n - 1$ changes of sign in Q. In this article the author writes a simple characterization for the best copositive approximation for elements of $C(Q)$ by elements of finite dimensional Strong Chebyshev subspace M of $C(Q)$. The result are given when Q is any compact subset of real numbers. He also shows that this best copositive approximation is unique. At the end of this article the author applies this result for different types of subsets Q of the real number.

Total Degree Rational Approximation [C-10B]

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Total degree rational approximation is the uniform approximation of discrete data by the set of rational functions whose numerator and denominator degrees sum to a fixed number called the total degree. Finding the best total degree rational approximation that exists is reported in Kemp [Nondegenerate Rational Approximation, *Approximation Theory X*, C. Chui, L. Schumaker, J. Stöckler (eds.), Vanderbilt University Press, Nashville, TN, (2002), 246-266]. Here we make significant refinements to its theory and algorithm. A new theorem states if the algorithm's output satisfies a certain simple condition, then the best rational approximation of total degree is also the best for all lesser degrees. The following refinements of the algorithm make it robust. 1. In the search for eigenvalues/eigenvectors, those at equal bounds are calculated without inverse iteration before those that fall between unequal bounds enabling the algorithm to handle numerically difficult cases like approximating a "discrete" sign function. 2. At most one guess is needed to find an eigenvalue by inverse iteration. 3. The Remes one point exchange swaps out every point in the reference set instead of one or two in order to increase the chance of finding the best total degree approximation by visiting more references. 4. The next reference set is chosen if it's pole free and has the greatest minimax error exceeding the previous minimax error. 5. The algorithm always finds the best minimax error for the total degree polynomial. 6. Examples validate the improved algorithm. Note that the total degree algorithm always produces approximations that exist (i.e satisfy Chebshev's equioscillation error criterion). Better ones may exist but no best one.

Dual Bases in Subspaces [C-10B]

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We discuss the construction of bases of functions in subspaces Y of a (finite-dimensional) linear space X that are dual to certain certain subsets of X^*. Or, in inner product spaces, we construct dual basis functions in one subspace Y_1 that are dual to bases in a second subspace Y_2. The constructions and properties are applied to subspaces of polynomials spaces in the B-form.

On the Involute D-scroll in the Euclidean 3-space $\mathbb{E}^3$ [C-13A]

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In an earlier paper, the author introduced the properties of the B-scroll in Euclidean 3-space $\mathbb{E}^3$. The B-scroll in the Minkowskean 3-space was also examined there. Deriving curves based on other curves is a subject in geometry. Involute-evolute curves, Bertrant curves are of this kind. In this paper, we consider two special ruled surfaces associated to a space curve α with curvature $k_1 \neq 0$ and involute of α is β. We define the Involute D-scroll of any curve as a ruled surface. We examine the positions of the D-scroll and the Involute D-scroll relative to each other.

Rate of Convergence for Generalized Baskakov Type Operators with Derivatives of Bounded Variation [C-13B]

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In the present paper, we estimate the rate of convergence for generalization of the Baskakov type operators with derivatives of bounded variation.

Reduced Spline Base Method for Computing Dimension of Multivariate Spline Spaces [C-10B]

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It is well-known that the spaces $S^r_d(\Delta_n)$ of C^r splines of degree d in n variables over a simplicial partition Δ_n can be treated as a (graded) module over the $(n+1)$-variate polynomial ring $\mathcal{P}_{n+1}$. In many cases, such modules can be completely analyzed using methods of algebraic geometry. For a fixed r, such analysis yields formulas for the dimension of $S^r_d(\Delta_n)$ for all degrees d; more importantly, it provides a (small) subset $B^r \subset S^r_d(\Delta_n)$ of spline functions such that any spline in $S^r_d(\Delta_n)$ is equal to a $\mathcal{P}_{n+1}$-linear combination of the elements in B^r. Such sets are called the reduced spline bases. For example, we show that, for any r and for any degree d, the number of elements in the reduced spline basis for the spline space on a rectangular grid on $\mathbf{R}^n$ is equal to the number of cells in the partition. In addition, all the elements in the reduced basis are tensor products of univariate splines. This expands a classical result of de Boor.

Approximate Solution of the One Dimensional Diffusion Equation within the Fixed Limits [C-18A]

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A model for the approximate solution of diffusion process is presented through this work and an attempt is also made for the applicability of Greens function method for solving the one dimensional diffusion equation within the desired limits. From this process we will get the required solution to this diffusion equation by considering the initial condition $t = 0$. This equation describes the rate of change of concentrations of substances to its own lattice or may be in different substances with a constant diffusion coefficient. At last a computational approach will also be used for getting the required approximate solutions. While solving the equation we throughout consider $t = 0$, so that the result may also be applicable in an isothermal diffusion.

Parabolic Molecules: Curvelets, Shearlets, and Beyond [P-2]

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Anisotropic representation systems such as shearlets and curvelets have had a significant impact on applied mathematics in the last decade. The main reason for their success is their superior ability to optimally resolve anisotropic structures such as singularities concentrated on lower dimensional embedded manifolds, for instance, edges in images or shock fronts in solutions of transport dominated equations. By now, a large variety of such anisotropic systems has been introduced, among which we mention second generation curvelets, bandlimited shearlets and compactly supported shearlets, all based on a parabolic dilation operation. These systems share similar approximation properties, which is usually proven on a case-by-case basis for each different construction. In this talk we will first introduce shearlets and discuss their sparse approximation properties. Based on this, we will then introduce the novel concept of parabolic molecules which allows for a unified framework encompassing all known anisotropic frame constructions based on parabolic scaling. The main result essentially states that all such systems share similar approximation properties. One consequence we will discuss is that at once all the desirable approximation properties of shearlets can be deduced for virtually any other system based on parabolic scaling.

Sparse Representations and Singularity Detection using Directional Multiscale Representations [M-1A]

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Several advanced multiscale systems, such as curvelets and shearlets, were introduced during the last decade to overcome the limitations of wavelets and other traditional representations. Shearlets, in particular, are designed to combine multiscale analysis through the framework of affine systems with the ability to handle directional information efficiently. As a result, they provide optimally efficient representations, in a precise sense, for a large class of 2D/3D data. The sparse approximation properties of shearlets are closely related to their ability to provide a precise geometric characterization of singularities. These observations point out to a wide range of image and data analysis applications where the shearlet framework can provide further insight.

On Nonconvex Minimization Approaches for Sparse Solutions [M-1B]

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I shall present some recent results on nonconvex minimization approaches for compressed sensing, ℓ_p minimization for $0 < p < 1$, the capped ℓ_1 minimization, and log-sum minimization, ℓ_1 greedy algorithm and CWB algorithm. Convergences of these algorithms will be discussed.

Construction of 3D Macro-Element on Alfeld's Split [M-8B]

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We shall present a construction of C^r macro-element over Alfeld's split in the trivariate setting. The construction extended the Alfeld and Schumaker's construction in 2005 from C^2 to C^r for $r \geq 3$. We first explain the degree of splines for this construction is minimal. Then we explain two kinds of new degrees of freedom in order to determine the elements when $r \geq 3$. The dimensions of the spline spaces associated with these C^r macro-elements is determined.

A Galerkin Radial Basis Function Method Applied to the Schrödinger Equation [M-8A]

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The simulation of quantum dynamical processes by the time-dependent Schrödinger partial differential equation (TDSE) reveals the dynamics of chemical bonds and reactions. Two main challenges when numerically solving the TDSE are the curse of dimensionality and the unboundedness of the domain. We consider the discretization of the TDSE using radial basis functions (RBF), which are easy to use also in high dimensions. We formulate the discretized problem over the unbounded domain without imposing explicit boundary conditions. By employing an RBF-Galerkin formulation, we can ensure time-stability together with conservation of total probability (the norm of the solution). The main part of the probability mass is localized to a small region in space. This is where we place the node points for the RBFs. Hence, the approximation error roughly consists of two parts; the interior error representing how well the RBFs approximate the solution function and the exterior error representing the part of the solution that we are neglecting. For Gaussian RBFs, we have shown theoretically that the error has exponential convergence down to to the level of the exterior error for solutions in the native space. How to select the method parameters in order to achieve good results has been studied numerically. Compared with a Fourier method with periodic boundary conditions in two dimensions, the RBF-Galerkin method yields more accurate results for the same number of discretization points.

Sparse Approximations of Piecewise C^β Functions in $L^2(\mathbb{R}^3)$ with C^α Singularities using Shearlets [M-1A]

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In this talk, we consider sparse approximations of a generalized model of cartoon-like images comprising of piecewise C^β functions that are smooth apart from C^α discontinuity edges. We show that compactly supported shearlet frames satisfying weak decay, smoothness, and directional moment conditions provide almost optimally sparse approximations of this generalized cartoon-like images class for the smoothness range $1 < \alpha \leq \beta \leq 2$.

The Construction and Analysis of Variational Integrators [M-16B]

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Variational integrators are a class of geometric structure-preserving numerical methods that are based on a discrete Hamilton's variational principle, and are automatically symplectic and momentum preserving. We will review the role of Jacobi's solution of the Hamilton–Jacobi equation in the variational error analysis of variational integrators, and demonstrate how it leads to two systematic methods for constructing variational integrators. In particular, Jacobi's solution can be characterized either in terms of a boundary-value problem or variationally, and these lead to shooting-based variational integrators and Galerkin variational integrators, respectively. Computable discrete Lagrangians can be obtained by choosing a numerical quadrature formula, and either a finite-dimensional function space or an underlying one-step method. We prove that the resulting variational integrator is order-optimal, in that the order of the resulting variational integrator is only limited by the order of accuracy of the numerical quadrature formula, and either the approximation properties of the finite-dimensional function space or the order of accuracy of the underlying one-step method. Furthermore, when spectral basis elements are used in the Galerkin formulation, one obtains geometrically convergent variational integrators.

Sparsity-inducing Dual Frames and Applications [M-5A]

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A notion of sparsity-inducing dual frames (sparse duals) for a given non-exact frame and its applications will be presented. Beside discussions of definitions and basic properties of sparse duals, a sparse-dual-based ℓ_1-analysis approach to compressed sensing problems and its performance analysis will be outlined. Examples will be provided.

Analysis-suitable T-splines [M-11A]

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Isogeometric methods rely on analysis-suitable geometric descriptions. These geometric descriptions should possess basic mathematical properties important for analysis such as linear independence and optimal approximation properties. In this talk we briefly review the emerging mathematical theory of analysis-suitable T-splines. Analysis-suitable T-splines are a generalization of NURBS that permit local refinement and the watertight representation of complex geometry of arbitrary topological genus.

Sparse Subsampling for Cartoon-like Images [M-1A]

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In this talk, we will present a novel sparse subsampling technique for piecewise smooth images which can be sparsely represented by directional systems. In particular, we will show our scheme can sparsely approximate such images f from the collection of samples corresponding to the Fourier coefficients of f with almost optimal sampling rate.

Best Onesided Approximation with Hermite-Biehler Weights [C-13B]

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For a given real-valued function f and a Hermite-Biehler function E, the onesided L-approximation problem with weight $|E|^{-2}$ asks for an entire function F of exponential type $\delta > 0$ such that $F(x) \geq f(x)$ for real x and the integral

$$\int_{-\infty}^{\infty} \{F(x) - f(x)\} |E(x)|^{-2} dx$$

is as small as possible. Such a minimizing function F is called an *extremal majorant* if it exists. Using a series representation for L^2-norms in the de Branges space $\mathcal{H}(E)$, we show that F is the extremal majorant for f provided $F(x) \geq f(x)$ for all real x and $F(\xi) = f(\xi)$ for all solutions of $B(\xi) = 0$ where $B = i/2(E - E^*)$. This framework can be used to construct explicit extremal majorants for functions of the form

$$f(x) = \int_{-\infty}^{\infty} e^{-\lambda|x|} d\nu(\lambda)$$

in $L^1(|E(x)|^{-2}dx)$. The Hermite-Biehler functions

$$E_r(z) = \Gamma(r+1)(z/2)^{-r}(J_r(z) - iJ_{r+1}(z))$$

with the Bessel functions J_r for $r > -1$ are of particular interest since they allow to extend these results to radial functions in higher dimensions.

On Sparse Solutions of Under-determined Linear Systems and Phase Retrieval [M-1B]

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We discuss the recoverability conditions for the multiple measurement vectors problem in under-determined systems (joint with M. J. Lai), low rank matrix recovery, phase retrieval (joint with Y. Wang), and its various generalizations. For vector recovery via numerically erasure-robust frames, we show the connection between the constructions of numerically erasure-robust frames and a geometry problem (joint with Y. Wang), and also discuss some progress on solving the problem if time allows.

Polynomial Splines over Locally Refined Box-Partitions [M-11A]

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For multivariate splines on regular partitions there are several ways to break the tensor product grid structure. In this talk we consider LR-splines which are polynomial splines over locally refined box partitions using tensor product B-splines as a basis. LR-splines are related to hierarchical B-splines introduced in 1988 by Forsey and Bartels, and to Sederberg's T-splines. Our motivation comes from isogeometric analysis where one uses the same basis both for modeling the geometry and for finite element calculations.

Adaptive Smolyak Pseudospectral Approximation [M-11B]

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Polynomial approximations of computationally intensive models are central to uncertainty quantification. A pseudospectral approach to high-dimensional polynomial approximation, based on Smolyak's algorithm with sparse grids, has recently been proposed by [Constantine *et al.* 2012]. We analyze this approach and establish results on the accuracy of Smolyak pseudospectral approximation, showing that the Smolyak approximation avoids "internal" aliasing and makes more effective use of sparse function evaluations than other approaches. These results are applicable to broad choices of quadrature rule and to generalized/anisotropic sparse grids. Exploiting this flexibility, we introduce several greedy heuristics for adaptive refinement of the pseudospectral approximation. We illustrate the accuracy and efficiency of an adaptive approach on various model problems and on a realistic chemical kinetics problem involving uncertain rate parameters.

Optimal Packings of Congruent Circles on Spheres and Flat Tori [M-3B]

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We consider packings of congruent N circles on spheres (the Tammes problem) and square flat tori. Toroidal packings are interesting due to a practical reason - the problem of super resolution of images. We classified all locally optimal spherical arrangements up to N=12. For a flat square torus we have found optimal arrangements for N = 6, 7 and 8. Our proofs are based on computer enumerations of spherical and toroidal irreducible contact graphs.

Bernstein's Inequality on Subsets of the Unit Circle [C-18B]

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In this talk we present new Bernstein-type inequalities for polynomials on subsets of the unit circle. The pointwise estimates use normal derivatives of Green's functions and the density of equilibrium measures. It is also shown that the estimates are sharp. Joint work with V. Totik.

Hierarchical Interpolation of Parameterized Functions [M-11B]

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We present a novel method for construction of polynomial interpolation grids on arbitrary Euclidean geometries on which there is a probability density. The interpolation nodes are defined as the solution to an optimization problem involving polynomials orthogonal under the density function. The interpolation process is a sequence and so nested refinement strategies can be employed. Finally, they are applicable for high-dimensional spaces. This construction of these grids is accomplished by an application of the Least Orthogonal Interpolant to the formation of weighted Leja sequences.

Meshless Galerkin Methods, Kernels and Quadrature [M-6A]

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In this talk we will discuss kernel-based Galerkin (weak) methods for numerically solving elliptic PDEs on the 2-sphere. These methods make heavy use of new, computable, rapidly decaying, "small-footprint" Lagrange bases and associated kernel quadrature formulas that were recently developed by E. Fuselier, T. Hangelbroek, N., X. Sun, J. D. Ward, and G. Wright.

Fast Ewald Summation under Mixed Boundary Conditions based on NFFTs [M-8A]

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We propose a new approach for the efficient calculation of the Coulomb interaction energy of charged particle systems subject to mixed boundary conditions. The electrostatic energy of N charges $q_j \in R$ at positions $\mathbf{x}_j \in R^3$, $j = 1, \dots, N$, is basically a sum of the form

$$\frac{1}{2} \sum_{i,j=1}^{N} \sum_{\mathbf{n} \in \mathcal{S}}{}' \frac{q_i q_j}{\|\mathbf{x}_i - \mathbf{x}_j + B\mathbf{n}\|},$$

where the index set $\mathcal{S} \subseteq Z^3$ is chosen according to the given boundary conditions and $B \in R$ is the edge length of the periodically dublicated simulation box. The prime on the infinite sum indicates that in the case $\mathbf{n} = \mathbf{0}$ all terms with $i = j$ are omitted. The well known Ewald summation formulas, which have at first been derived for the fully periodic case, i.e., $\mathcal{S} := Z^3$, and are valid if charge neutrality is assumed, are the principle behind many fast algorithms evaluating the electrostatic interaction energy of charged systems. Thereby the poorly converging sum is split into two exponentially fast converging parts, where a precise order of summation has at first to be specified. The long range part can then be written as an absolutely converging sum in Fourier space. In the fully periodic case, the Fast Fourier transform can be applied to trim the computational costs to $\mathcal{O}(N \log N)$ arithmetic operations, which is the basic idea behind so called Particle Mesh approaches as well as algorithms based on nonequispaced fast Fourier transforms (NFFTs). For open boundary conditions, i.e. $\mathcal{S} := \{0\}^3$, fast summation methods based on NFFTs where suggested, too. In this talk we aim to close the gap and suggest NFFT based algorithms also for 2d- and 1d-periodic boundary conditions, i.e., $\mathcal{S} := Z^2 \times \{0\}$ and $\mathcal{S} := Z \times \{0\}^2$, respectively.

Decomposition Type Smoothness Spaces [M-1A]

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In applicable harmonic analysis, smoothness space are often designed following the principle that smoothness should be characterized by (or at least imply) some decay or sparseness of an associated discrete expansion. A well known example is that sparse orthonormal wavelet expansions correspond to smooth functions measured on the Besov scale. In this talk I will give an overview of a fairly general method to construct both isotropic and anisotropic smoothness spaces on R^d based on structured decompositions of the frequency space. The resulting smoothness spaces have many desirable properties. For example, adapted frames for L_2 with compact support can be constructed, and the smoothness norm can be completely characterized by a sparseness condition on the frame coefficients.

Spectral Convergence for Orthogonal Polynomials on Triangles and their Application on Hyperbolic Conservation Laws [M-16A]

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We study the behavior of orthogonal polynomials on triangles and their coefficients in the context of spectral approximations of partial differential equations. In these spectral approximations one studies series expansions $u = \sum_{k=0}^{\infty} \hat{u}_k \phi_k$ where the ϕ_k are orthogonal polynomials. We show that for any function $u \in C^\infty$ the series expansion converges faster than with polynomial order. With this result we are able to use these orthogonal polynomials in the super spectral vanishing viscosity method (SSV) in the context of conservation laws. We show the connection between the SSV-procedure and filtering. Finally we build from the differential operator expontential filters and consider some numerical test- cases.

Analysis of Normal Multiscale Transforms [M-18B]

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Normal multiscale transforms create meshes $v^j = Sv^{j-1} + d^j \cdot \hat{n}^j$ on a given two-dimensional surface Σ in $\mathbf{R}^3$ by predicting a base mesh $\hat{v}^j = Sv^{j-1}$ by applying a (linear) subdivision operator S, and an associated set of approximate normals $\hat{n}^j$ from the coarse mesh v^{j-1}. The points in v^j are then determined by intersecting the resulting approximate normal lines with Σ, and storing the scalar offsets into the Detail sequences d^j. These nonlinear transformations have been proposed for reparameterization and compression of surfaces meshes, and also used in image analysis, and front-tracking applications. We summarize our results on the analysis of convergence, detail decay, and Limit smoothness of normal multiscale transforms for Hölder-smooth surfaces Σ and meshes without extraordinary vertices, and provide numerical evidence confirming the theoretical findings.

Kolmogorov Type Inequalities for Fractional Derivatives of Multivariate Functions [C-15B]

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Let $L^{\Delta}_{\infty,\infty}(R^m)$ be the spaces of functions $f \in L_\infty(R^m)$, such that $\Delta f \in L_\infty(R^m)$.
In this talk, we shall present new sharp Kolmogorov type inequalities estimating uniform norm of fractional derivatives of a function $f \in L^{\Delta}_{\infty,\infty}(R^m)$ with the help of uniform norm of f and uniform norm of Δf. We shall discuss some applications of the resulting inequalities.

One-bit Matrix Completion [M-6B]

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Let Y be a matrix representing senate voting results in which each entry is either 1 for a vote of "yes" or -1 for a vote of "no". Now suppose that a number of entries are missing from Y (for example, senators may be out of town during a vote). Could you guess how to fill in the missing entries (how would senator i have voted on bill j)? Similar questions arise in many other applications such as recommender systems or binary survey completion. In this talk, we assume that the binary data is generated according a probability distribution which is parameterized by an underlying matrix M. Further, we assume that M has low rank; loosely, this means that the voting preferences of each senator may be parameterized by just a few characteristics (Democrat, Republican, etc.), although these characteristics need not be known. We show that the probability distribution of the missing entries of Y may be well approximated using maximum likelihood estimation under a nuclear-norm constraint. Under appropriate assumptions, we demonstrate that the approximation error is nearly minimax. The upper bounds are proven using techniques from probability in Banach spaces. The lower bounds are proven using information theoretic techniques.

Geometric Separation Using Shearlets Application to Road Line Extraction [C-5A]

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In this talk, we will discuss an application of Shearlets to the road extraction task from remote sensing. Utilizing the geometric separation property, 1-D discontinuities which correspond to road structures can be extracted from the given 2-D digital image. Hybrid Wavelet and Shearlet schemes are studied as well. We present some experimental results and a preliminary analysis is given to highlight the advantages of using Shearlets based schemes compared with variational BV-L^2 or BV-L^1 image decomposition approaches.

Multivariate Trigonometric Wavelets [C-10A]

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In this talk we generalize one-dimensional shift-invariant spaces of trigonometric kernels of de la Vallée Poussin type to multivariate shift-invariant spaces on non-tensor product patterns. For a $(d \times d)$-integer matrix M with $|\det M| > 1$ an element of the shift-invariant space spanned by a trigonometric kernel function φ is given as the linear combination $\sum_{\ell \in \Gamma} c_\ell \varphi(\cdot - 2\pi M^{-1}\ell)$, where Γ denotes the full collection of coset representatives of Z^d/MZ^d. Decompositions of these shift-invariant spaces are given by divisibility considerations on M. For these spaces of trigonometric polynomials we discuss the dimension and we construct interpolatory and orthonormal bases. The results are applied to construct an adaptive multiresolution on the Hilbert space $L^2(T^d)$. Different matrices M can be used to obtain anisotropic wavelet spaces. This is joint work with R. Bergmann (Lübeck) and D. Langemann (Braunschweig).

Kernels for Parametric Operator Equations [M-8A]

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In the study of parametric operator equation as they occur for instance in the recent field of uncertainty quantification, one often implicitly encounters reproducing kernels. It is well known that the eigenvalue decay of the associated Carlemann operator determines the regularity of the solution to the operator equation with respect to the parameter. The regularity is in most cases analytical and can be modeled via reproducing kernel Hilbert spaces of non-standard kernels. We will provide examples for such kernels and discuss their approximation properties.

Orthogonal Parameterized Wavelets and Pattern Matching [C-10A]

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The design of the most popular discrete orthgonal wavelets typically imposes a certain number of vanishing moments as well as a mimimal phase requirement. These properties restrict the flexibility of the resulting wavelet giving the researcher only a handful of orthogonal wavelets to employ. By starting with just the orthogonality condition, a necessary and sufficient parameterization can be constructed for the entire class of discrete orthogonal wavelets with dilation factor 2 for a given length. This results in a continuum of orthgonal wavelets of varying smoothness, shape, and regularity. In this talk I will show how the length ten parameterization contains wavelets which can be chosen to match a certain frequency response or pattern, and suggest potential applications of this matching ability.

Chebyshev-Grüss-Type Inequalities: A New Approach [C-13B]

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The classical form of Grüss' inequality gives an estimate of the difference between the integral of the product and the product of the integrals of two functions in $C[a, b]$. The aim of this talk is to introduce a new Chebyshev-Grüss-type inequality and apply it to the well-known Bernstein and Szász-Mirakjan operators. Conjectures regarding these operators are given. Some examples are also presented, in order to underline the advantages of the new approach.

Quantization and Encoding for Oversampled Signals [M-6B]

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Analog-to-digital (A/D) conversion is the process by which signals (viewed as vectors) are replaced by bit streams to allow for digital storage, transmission, and processing using modern computers. Typically, A/D conversion is thought of as being composed of sampling and quantization. Sampling consists of collecting inner products of the signal with appropriate (deterministic or random) vectors. Quantization consists of replacing these inner products with elements from a finite set. Often, quantization is followed by compression or encoding, in order to reduce the size of the digital data. A good A/D scheme allows for accurate reconstruction of the original object from its quantized (and compressed) samples. In this talk, we discuss methods for quantizing and encoding oversampled signals. We focus on finite dimensional signals and sparse signals. We employ Sigma-Delta quantization, and investigate the reconstruction error as a function of the bit-rate.

Analysis-Suitable and Dual-Compatible T-splines [M-11A]

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We now introduce the concept of *Dual-Compatible (DC) T-splines*, and, correspondingly, *Dual-Compatible T-meshes.* A T-mesh is DC if, given any two blending functions B_i and B_j constructed on it, they admit a common underlying knot vector at least in one coordinate direction. This property is relevant because on a DC T-mesh it is easy to construct suitable functionals $\lambda_i(\cdot)$ which are dual to the blending functions, i.e., $\lambda_i(B_j) = \delta_{i,j}$, where B_j denotes a generic DC T-spline blending function and $\delta_{i,j}$ is the usual Kronecker symbol. The existence of such dual functionals, which then form a dual basis, provides the space of T-splines with a rich mathematical structure which brings several interesting consequences, such as the linear independence of blending functions, the partition of unity property (provided that the constant function belongs to the space and thanks to the structure of the dual functionals we will introduce). The dual basis is used to define a projector (onto the space of DC T-splines) which serves as a key ingredient for the analysis of the approximation properties of DC T-spline spaces. Finally, we show that DC T-splines are equivalent to Analysis Suitable T-splines.

Nonlinear Approximations for Subspace Segmentation [M-3A]

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Given a set of data $\mathbf{W} = \{w_1, ..., w_N\} \in R^D$ drawn from a union of subspaces, we focus on determining a nonlinear model of the form $\mathcal{U} = \bigcup_{i \in I} S_i$, where $\left\{S_i \subset R^D\right\}_{i \in I}$ is a set of subspaces, that is nearest to $\mathbf{W}$. The model is then used to classify $\mathbf{W}$ into clusters. Our approach is based on the binary reduced row echelon form of data matrix, combined with an iterative scheme based on a non-linear approximation method. We prove that, in absence of noise, our approach can find the number of subspaces, their dimensions, and an orthonormal basis for each subspace S_i. We provide a comprehensive analysis of our theory and determine its limitations and strengths in presence of outliers and noise.

Spectral Analysis and Optimal Iterative Methods for IgA Linear Systems [M-16A]

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We study the spectral properties of stiffness matrices that arise when isogeometric analysis is employed for the numerical solution of classical second order elliptic problems. Motivated by the applicative interest in the fast solution (by preconditioned Krylov or multigrid methods) of the related linear systems, we look for a spectral characterization of the involved matrices. In particular, we investigate non-singularity, conditioning (extremal behavior), spectral distribution in the Weyl sense, as well as clustering of the eigenvalues to a certain (compact) subset of the complex field. All the analysis is related to the notion of symbol in the Toeplitz setting and is carried out both for the cases of 1D and 2D problems. The spectral properties represent the starting point for designing fast two-grid methods for which we provide a numerical confirmation of the optimality, meaning that the spectral radii of the related iteration matrices are bounded by a constant c_p for all n, $c_p < 1$: a formal proof of optimality for $p = 2$ and $p = 3$ is given An extension of the results to the two-level case is provided, together with a wide set of numerical tests including the V-cycle and the W-cycle applied to approximated 1D and 2D problems. Joint work with C. Garoni, C. Manni, F. Pelosi, and H. Speleers

Dimension of C^2 Trivariate Splines on Cells [M-8B]

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Alfeld, Schumaker and Whiteley (1993) determined the generic dimension of the space of C^1 splines of degree $d \geq 8$ on tetrahedral decompositions. In this paper, we analyze the dimension of C^2 trivariate splines on cells, which are tetrahedral complexes sharing a single interior vertex. The dimension depends on subtle geometry of the fatpoints corresponding to the configuration of the hyperplanes adjacent to the interior vertex. A key tool is the classification of the relevant fatpoint ideals by Geramita, Harbourne and Migliore(2009).

From 4-point to Bernstein: the Adaptation of Approximation Operators to Positive Definite Matrices [M-16B]

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The class of symmetric positive definite (SPD) matrices is important both in theory and application. We introduce the concept of admissible matrix mean and apply it for data consisting of SPD matrices. We suggest to use this approach to adapt approximation methods for SPD-valued data. In particular, we illustrate this adaptation for the interpolatory 4-point subdivision schemes, the corner cutting subdivision schemes, and Bernstein operators for the approximation of SPD-valued functions. This is a joint work with Uri Itay, under the supervision of Nira Dyn.

Regularity of Multivariate Birkhoff Interpolation Schemes [C-18A]

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I will present a solution to a conjecture of Ron-Qing Jia and A. Sharma regarding regularity of certain multivariate Birkhoff interpolation schemes. While originally conjectured for polynomials over reals, the conjecture turns out to be true over the complex field and false over the real field.

Blind One-Bit Compressive Sampling [M-1B]

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In this talk, we introduce an optimization model for reconstruction of sparse signals from 1-bit measurements. The model targets a solution that has the least l0-norm among all signals satisfying consistency constraints stemming from the 1-bit measurements. An algorithm for solving the model is developed. Convergence analysis of the algorithm is presented. Our approach is to obtain a sequence of optimization problems by successively approximating the l0-norm and to solve resulting problems by exploiting the proximity operator. We examine the performance of our proposed algorithm and compare it with the binary iterative hard thresholding (BIHT) for 1-bit compressive sampling reconstruction. Unlike the BIHT, our model and algorithm does not require a prior knowledge on the sparsity of the signal. This makes our proposed work a promising practical approach for signal acquisition.

Exact Asymptotics of Best Adaptive Asymmetric Approximation of Bivariate Convex Functions by Piecewise-Linear Splines [M-5B]

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In this talk we shall present new results on exact asymptotic behavior of the error of the best adaptive asymmetric approximation of bivariate functions with non-negative Hessian by piecewise-linear splines. We shall also discuss extensions of these results to higher dimensions.

Maximal Hyperplanes of ℓ_p^n with Respect to Relative Projection Constant [C-15A]

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We will investigate the problem of finding the hyperplane H in ℓ_p^n that maximizes relative projection constant $\lambda(H, \ell_p^n)$. That is we are trying to find the hyperplane H that has the biggest minimal projection among all minimal projections onto hyperplanes of ℓ_p^n.

Imposing Angle Boundary Conditions on B-spline Surfaces [C-13A]

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In this talk, we focus on the problem of modeling a B-spline surface with the prescribed angle distribution along a given boundary B-spline curve of this surface. This study is motivated by a modeling of the Pelton turbine bucket, especially its inner surface, where the most difficult part is to satisfy the given angle distributions along an outlet curve and a splitter which are boundary curves of a B-spline surface representing the inner surface of the bucket. We study the existence of an exact solution of this problem, i.e., under which conditions on a given boundary curve and/or prescribed angle distribution a tangent ruled surface of a resulting B-spline surface can be found. It turns out that such conditions are very strict and an exact solution exists only in very special cases. Thus, we formulate an algorithm for finding an approximate solution of this problem, study its approximation order and derive a bound on the approximation error. Finally, the proposed method is demonstrated on examples.

Intrinsic Supersmoothness of Bivariate Splines [M-8B]

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We show that many spaces of bivariate splines possess additional smoothness (supersmoothness) that is not reflected in their definitions. Smoothness is treated as vanishing of strongly supported smoothness functionals. We investigate the dependence of the phenomenon of supersmoothness on the geometry of the underlying partition and the degree of splines. In particular, we prove that bivariate splines of sufficiently low degree cannot have different smoothness across collinear edges in the underlying cell. Moreover, such splines do not have non-collinear edges in the cell, that is, such edges can be removed.

Beyond Tensor-product Structures in IgA: the PS Perspective [M-11A]

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Isogeometric Analysis (IgA) is now a well established method for the analysis of problems governed by partial differential equations. Its main goal is to improve the connection between numerical simulation and Computer Aided Design (CAD) systems. The same functions, usually tensor-product B-splines or NURBS, are used both to describe the geometry and to approximate the unknown solutions of differential equations. A key ingredient in IgA is to retain along the entire analysis process the exact geometry of the physical domain. In this talk we consider the use of quadratic Powell-Sabin (PS) splines as an alternative tool in IgA. They are C^1 piecewise quadratic polynomials defined on the so-called PS refinement of any given triangulation. They can be represented with basis functions possessing similar properties to the classical tensor-product B-splines. The so-called PS B-splines form a convex partition of unity, and the coefficients of this representation have a clear geometric meaning. A rational extension of PS splines, referred to as NURPS surfaces, can also be easily defined. NURPS surfaces allow an exact representation of quadrics, and their shape can be locally controlled by control points and weights in a geometrically intuitive way. Thanks to their structure based on triangulations, PS/NURPS splines offer the flexibility of classical Finite Element Methods (FEM) with respect to local refinements. Moreover, they share with standard tensor-product NURBS the increased smoothness and the B-spline like basis. Therefore, they constitute a natural bridge between classical FEM and NURBS-based IgA. In addition, they are also well suited to represent more complex geometries beyond a quadrilateral topology.

The Eigenstructure of Operators linking the Bernstein and the Genuine Bernstein-Durrmeyer Operators [C-13B]

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We study the eigenstructure of a one-parameter class of operators U_n^ϱ of Bernstein-Durrmeyer type that preserve linear functions and constitute a link between the so-called genuine Bernstein-Durrmeyer operators U_n and the classical Bernstein operators B_n. In particular, for $\varrho \to \infty$ (respectively, $\varrho = 1$) we recapture results well-known in the literature, concerning the eigenstructure of B_n (respectively, U_n). The last section is devoted to applications involving the iterates on U_n^ϱ.

Convolution Stability for Signals with Finite Rate of Innovation [M-3A]

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In this talk, I will discuss universal necessary conditions for stable recovery of signals with finite rate of innovation. This talk is based on a joint work with Jun Xian.

Sampling Scattered Data with Bernstein Polynomials [M-6A]

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Viewing the classical Bernstein polynomials as sampling operators, we study a generalization by allowing the sampling operation to take place at scattered sites. We utilize both stochastic and deterministic approaches. On the stochastic side, we consider the sampling sites as random variables that obey some naturally derived probabilistic distributions, and obtain Chebyshev type estimates. On the deterministic side, we incorporate the theory of uniform distribution of point sets (within the framework of Weyl's criterion) and discrepancy method. We establish convergence results and error estimates under practical assumptions on the distribution of the sampling sites.

Approximation Properties on Singularly Parametrized Domains in Isogeometric Analysis [M-16A]

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Isogeometric analysis is a numerical method based on the NURBS representation of CAD models. In the standard approach the geometry parametrization of a 2-dimensional physical domain posesses the tensor product structure of bivariate NURBS. Hence the domain is structurally equivalent to a rectangle. The special case of singularly parametrized NURBS surfaces is used to represent non-quadrangular domains without splitting. We analyze the approximation properties of the isogeometric test function spaces on singular parametrizations and present local refinement strategies that lead to geometrically non-degenerating splittings of singular patches. Using this we develop a general framework to prove approximation results on singularly parametrized domains. We prove bounds for the L^2 and H^1 approximation error for two classes of singular parametrizations of two dimensional domains.

Bernstein Inequality in L^α Norms [C-15B]

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The classical Bernstein inequality estimates the derivative of a polynomial at a fixed point with the supremum norm and a factor depending on the point only. Recently, this classical inequality was generalized to arbitrary compact subsets on the real line. That generalization is sharp and naturally introduces potential theoretical quantities. It also gives a hint how a sharp L^α Bernstein inequality should look like. In this talk we present a proof of this conjectured L^α Bernstein type inequality and we also discuss its sharpness.

Voronoi and Voronoi-related Tessellations With Moving Data [C-18B]

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For many applications on communications, transportation and others, it is interesting to be able to apply voronoi tessellations and their uses to a moving set of points in order to know for example the set of natural neighbors of a given point at any time while it or all are in movement. We discuss the cases where all movements are known and when some of them are known and some others not. We use quaternionic analysis to do the above because it is simpler and faster to calculate movements and relative positions with it. We do it to both calculate the positions and the tessellation, defining for that the concept of voronoi tessellation on a quaternion environment

Christoffel Functions for Doubling Measures on Quasismooth Curves and Arcs [C-15B]

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We give two-sided estimates for Christoffel-functions associated with doubling measures on quasismooth curve and arc. Particularly, we get estimates for Dini-smooth curve and arc which involve previously results known on intervals. As applications we obtain estimates for orthogonal polynomials and Nikolsky-type inequalities.

Convolution Based Monotone Interpolation [C-18A]

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We consider a convolution based procedure for constructing monotone interpolants with continuous derivatives of higher order. The procedure is iterative and is used to obtain analytic functions with functional values exponentially close, with respect to the number of iterations, to the given interpolation values. Applications in signal processing are discussed.

On the Dimension of Splines on Tetrahedral Partitions [M-8B]

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We consider the linear space of globally differentiable piecewise polynomial functions defined on a three-dimensional polyhedral domain which has been partitioned into tetrahedra, and the problem of finding the dimension of such a space. We prove a lower and an upper bound applying homological techniques and exploring connections between splines and ideals generated by powers of linear forms, fat points and the Fröberg's conjecture. The formulas we present apply for any degree of the polynomials, any order of global smoothness, and include terms that explicitly depend on the number of different planes surrounding the edges and vertices of the partition. In some cases, they give better approximations to the exact dimension, and more importantly, the construction gives an insight into ways of dealing with the dimension problem.

Fourier-based Matching of Flexible Atomic Structures [C-18A]

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We discuss a generalized framework for flexible molecular fitting, where a high-resolution crystal structure of a molecule is deformed to optimize its position with respect to a low-resolution density map. During this task three questions arise: How can we describe the optimal position of molecule and map? How do we find it efficiently? How can the crystal structure be deformed to obtain the best match? The talk will start answering the first two questions by providing several useful definitions of the optimal fit between the crystal structure and the density map in terms of evaluating correlation integrals and by explaining the non-uniform rotationally exhaustive Fourier-based search scheme to solve the correlation problem over arbitrary subsets of rigid-body motions. Our scheme will take advantage of the fast Fourier transform on the rotation group to achieve a speedup with respect to the sought 5-dimensional rotation. To answer the remaining question, we will present a hierarchical domain-based flexibility model based on an iterative domain decomposition of the input protein. Since the rotational Fourier-based search scheme is capable of searching over arbitrary subsets of the space of rigid-body motions; we use this property to flexibly fit each domain of the molecular structure under its specific range of motion.

Steerable Wavelet Frames [C-10A]

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In this talk, we shall propose a method of constructing steerable wavelet frames of $L_2(R^d)$ that generalizes and unifies previous approaches, including Simoncelli's pyramid. Such wavelets can be constructed by decomposing each element of a wavelet frame into a finite collection of oriented wavelets. The key to this construction is that the decomposition is an isometry, whereby the new collection of wavelets maintains the frame bounds of the original one. The general method that we propose here is based on partitions of unity involving spherical harmonics. A fundamental aspect of this construction is that Fourier multipliers composed of spherical harmonics correspond to singular integrals in the spatial domain.

Function Interpolation via Weighted ℓ_1-minimization [M-6B]

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We consider the problem of interpolating a function from sample values. We take into consideration that functions often possess certain smoothness, such as that their low-order Fourier coefficients are more likely to appear in the best s-term approximation than high-order Fourier coefficients. Weighted ℓ_1 minimization turns out to be a natural recovery method for such smoothness assumptions. We will present theoretical results and promising numerical experiments which indicate that weighted ℓ_1 polynomial interpolations effectively suppresses spurious oscillatory artifacts that often plague least squares solutions without sacrificing accuracy. If time permits, connections to polynomial chaos expansions in the context of uncertainty quantification will also be discussed.

**Approximation on Surfaces with Kernels:
Recent Developments and Applications [P-17]**

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Kernel approximation methods, such as radial basis functions, are advantageous for a wide-range of applications that involve analyzing/synthesizing scattered data, or numerically solving partial differential equations on geometrically difficult domains. Although initially considered for Euclidean domains, there has been much interest in kernel approximation methods defined on more general mathematical objects, which include, among many others, smooth surfaces in $\mathbf{R}^d$. In the case of $d = 3$, these domains arise in many applications from the geophysical and biological sciences, as well as computer graphics. We survey some recent developments of kernel approximation on surfaces, including error estimates, stable bases, and scalable algorithms, in the context of applications to geophysical fluid dynamics, biological pattern formation, and quadrature. We conclude with some thoughts on promising future directions.

Block Coordinate Descent for Multi-convex Optimization [M-1B]

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This talk is about regularized block multi-convex optimization, where the feasible set and objective function are generally non-convex but convex in each block of variables. There are many practical examples such as nonnegative matrix factorization, sparse dictionary learning, and multi-way tensor decomposition. To solve these problems, we propose a generalized block coordinate descent method, which uses three different block-update schemes and lets the users freely choose one of them so as to make the algorithm most efficient. Based on the so-called Kurdyka-Lojasiewicz inequality, we give a global convergence of the method with an estimation of convergence rate. Finally, we will use some numerical experiments on sparse nonnegative matrix factorization and sparse dictionary learning to illustrate the efficiency of the algorithm.

Best Approximation by Polynomials on Spheres and Balls [P-14]

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The purpose of this talk is to give an overview on best approximation by polynomials on the unit sphere and the unit ball in $\mathbb{R}^d$. I will report fairly recent results that provide a satisfactory solution for characterizing the best polynomial approximation on the unit sphere by a new modulus of smoothness, defined in the rotational angles on (x_i, x_j) planes, and its equivalent K-functions, defined in terms of the infinitesimal operators $D_{i,j} = x_i \partial_j - x_j \partial_i$. The modulus of smoothness is essentially the maximum of a family of modulus of smoothness in one variable, which allows one to tap into the rich classical trigonometric approximation theory for studying approximation on the sphere. The results on the unit ball include two moduli of smoothness and K-functionals, both can be used to characterize the best polynomial approximation, one of which extends the Ditzian-Totik modulus of smoothness from $[-1, 1]$ to the unit ball in $\mathbb{R}^d$.

Solving Support Vector Machines in Reproducing Kernel Banach Spaces with Matérn Functions [M-8A]

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In this talk we show how to use the different Matérn functions to solve the support vector mathines in the reproducing kernel Banach spaces. Because the Matérn functions are the positive definite functions. We can set up the reproduction property in the generalized native spaces by Fourier transform techniques such that they become the reproducing kernel Banach spaces and their reproducing kernels are introduced by the Matérn functions. Given the training data sites, the representations of the optimal solutions of the support vector machines (regularized empirical risks) in these reproducing kernel Banach spaces are the explicit forms in terms of the Matérn functions and the data points, and moreover, their coefficients can be computed by the fixed point iteration method.

Smoothed and Parallel Sparse Optimization [M-1B]

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Sparse optimization has found interesting applications in many areas such as machine learning, signal processing, compressive sensing, medical imaging, etc. Sparse optimization involves minimizing the ℓ_1-norm or ℓ_1-like functions, which are non-differentiable. In order to apply the classic methods such as gradient descent and quasi Newton, a traditional trick is to consider smoothed approximates of the ℓ_1-norm such as the Huber-norm, $(1+\epsilon)$-norm, and the sum of $\sqrt{x_i^2+\epsilon}$, where $\epsilon > 0$ is a small parameter. They cause a (slight)loss in solution sparsity and often require tuning ϵ. This talk introduces an alternative "smoothing" approach that does not directly generate a smooth function but produces an unconstrained dual problem whose objective is differentiable and enjoys a "restricted" strongly convex property. Not only can one apply a rich set of classic techniques such as gradient descent, line search, and quasi-Newton methods to this dual problem, exact sparse solutions and global linear convergence is guaranteed. In addition, parallelizing the algorithm becomes very easy. Theoretically, this approach gives the first global linear convergence result among the gradient-based algorithms for sparse optimization, including the recent algorithms based on operator splitting (ISTA/FISTA), variable spitting (Bregman/ADMM), and/or coordinate descent, which converge sublinearly in the global sense. Numerical examples are presented on problems in compressive sensing.

First-order Methods for Convex Minimization
Better Rates under Weaker Conditions [M-6B]

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This talk first overviews the fundamentals of minimizing a differentiable convex function and then shows that the well-known convergence rates of gradient methods can be achieved under conditions much weaker than the commonly accepted ones. We relax the (gradient) Lipschitz-continuity and strongly-convex conditions to ones that hold only over certain line segments. Not only are these relaxed conditions satisfied by more functions, they also give better constants in the bounds. Therefore, this work extends the classes of functions with minimization complexity guarantees and improves the complexity bounds. We apply these results to augmented ℓ_1 minimization by showing its linear convergence with an improved rate and demonstrating a faster algorithm.

Differential Proximity Condition
for Manifold-Valued Data Subdivision Schemes [M-18B]

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Since the inception of the proximity method by Wallner and Dyn, the method remains the one and only one – and also very effective –method for the analysis of the regularity, among other, properties of manifold-data subdivision schemes. The proximity condition is a sufficient condition for telling when a nonlinear subdivision shares the same regularity of a nearby linear subdivision scheme. Why is this method so successful, leading not only to much interesting analysis of different manifold-valued subdivision schemes, but also guiding the developments of new schemes? We "answer" this question by showing that, in fact, the original proximity condition can be reformulated in a way that makes it (a) easier to use, (b) much easier to interpret, and (c) provably both necessary and sufficient for the so-called smoothness equivalence property. One key observation here is that a lack of proximity condition corresponds exactly to the presence of some sort of "resonance" in a discrete dynamical system underlying the nonlinear subdivision scheme; such a resonance slows down a certain decay of the system when compared to that of its linear part, which in turn leads to a breakdown of smoothness equivalence. While the original motivation of this new theory of Differential Proximity Condition is to solve the perplexing necessity question, an unexpected property is also discovered on the sufficiency side, leading to a resolution of a confusing – and somewhat embarrassing – state of affairs in the original theory.

Hermite Interpolation and Bezier-type Geometric Modeling via Smooth GERBS on Triangulations [C-13A]

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In this presentation we shall extend the scope of applications of the construction proposed in the talk of Lubomir Dechevsky at the conference. This construction is based on smooth generalized expo-rational B-splines (GERBS) on triangulations and regularized Taylor polynomials. Here we shall consider the non-regularized version of this construction (when the regularization kernels are Dirac delta-functions) for the case of Hermite interpolation of sufficiently smooth functions on scattered-point sets. Our purpose will be to conduct a study of the performance of this construction in comparison with the performance of Bernstein-Bezier triangular macroelements on Clough-Tocher and Powell-Sabin splits. Among the criteria for this comparison will be:

1. number of degrees of freedom needed to achieve equal rates of approximation via the Hermite interpolation process;
2. constant factors to these rates and performance on coarse meshes;
3. size and structure of the generalized Vandermonde matrix of the Hermite interpolation process
4. complexity and structure of the transformation of the Hermite-interpolatory form into Bezier form and vice versa;
5. complexity and structure of the geometric modeling process via modification of the coefficients of the Bezier form;
6. complexity of the sparsification process when the order of Hermite interpolation is variable over the scattered-point set;
7. parallelizability of the algorithms in items 4 and 5;
8. load balancing of the parallel versions of the algorithms in items 4 and 5 when the order of Hermite interpolation is constant over the scattered-point set.

Gabor Shearlets: Low Redundant Directional Multiscale Representation Systems with Optimally Sparse Approximation [M-1A]

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In this talk, we shall introduce Gabor Shearlet systems, a new type of directional multiscale representation systems which has many desirable properties such as low redundancy, tight frames, MRA structure, optimally sparse approximation, etc. Based on a point transformation having Jacobian of magnitude one $\gamma : \mathbb{R}^* \times \mathbf{R} \to \mathbb{R}^2$, and a unitary operator $\Gamma : L^2(\mathbb{R}^2) \to L^2(\mathbb{R}^2)$, we transform the construction of our new shearlet system to the construction of tensor product of two systems: One is a wavelet system along the radial direction, and the other is a Gabor system along the shear direction. As a consequence, under the point transformation, the system can be implemented with standard filters from wavelet theory in combination with standard Gabor windows. Unlike the usual shearlets, in addition to tightness, the new construction can achieve a redundancy as close to one as desired. In order to achieve optimally sparse approximation for Cartoon-like functions, we introduce a pair of complementary orthogonal projections P_h, P_v for $L^2(\mathbb{R}^2)$. The adaptation of these two projections to our systems give rise to a cone-adaptive Gabor shearlet system, which we can show to achieve optimally sparse approximation for cartoon-like functions.

Kernels from Spectral Decompositions and their Approximation Properties [M-8A]

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The performance of kernel-based reconstruction methods depends heavily on the choice of the kernel. Practical experience and theoretical results motivate to employ problem adapted kernels which take as much structural information of the underlying problem into account as possible. Typically, such kernels do not allow for closed form expressions, as they arise, for instance, from spectral decompositions of self-adjoint operators. In this talk, reconstruction schemes based on such problem-induced kernels and numerically feasible approximations are discussed, in particular convergence orders and stability properties.