LIST OF ABSTRACTS

Optimal Polynomial Interpolation of High-dimensional Functions [M-13A]

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Many problems in scientific computing require the approximation of smooth, high-dimensional functions from limited amounts of data. In sufficiently high dimensions, such functions are known to possess sparse expansions in suitable orthogonal polynomial bases. Hence, it is natural to seek to apply techniques from compressed sensing to approximate such functions from small amounts of data. In this talk, I will discuss a framework for high-dimensional interpolation using weighted l^1 minimization. A new approximation result for this problem will be introduced, valid for arbitrary choices of weights and orthogonal bases. Two consequences of this result are as follows. First, it allows one to identify a generically optimal choice of weights for a given basis so as to minimize the approximation error. For appropriate choices of support sets - specifically, lower sets - the corresponding condition on the number of measurements required is provably optimal and greatly reduces (or even removes) the curse of dimensionality. Second, if some prior knowledge about where the largest coefficients lie is available, this result shows the benefits of adapted weighting strategies based on this information. The advantage of such strategies have been shown numerically in a number of recent studies, and this work provides a theoretical basis for these results. This aside, this framework also overcomes some technical issues with previous results. In particular, it removes an unrealistic condition that requires the norm of the expansion tail be known in advanced or accurately estimated.

Perturbing Beyond a Quarter and Finite Points Oversampling [C-10B]

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In this talk we present the class of entire functions of exponential type that are bandlimited in distributional sense. To reconstruct this kind of generalized functions we consider oversampling. We will have oversampling in two ways. One way will be by scaling the integers and using the Paley-Wiener-Schwartz theorem and the other way by performing classical computation using contour method. We will see that the contour method helps to minimize the oversampling. Moreover, the perturbation that goes beyond a quarter from the integers will be discussed.

Standard Finite Elements for the Numerical Resolution of the Elliptic Monge-Ampère Equation [M-10A]

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We prove a convergence result for a natural discretization of the Dirichlet problem of the elliptic Monge-Ampère equation using finite dimensional spaces of piecewise polynomial C^1 functions. Discretizations of the type considered in this paper have been previously analyzed in the case the equation has a smooth solution and numerous numerical evidence of convergence were given in the case of non smooth solutions. Our convergence result is valid for non smooth solutions, is given in the setting of Aleksandrov solutions, and consists in discretizing the equation in a subdomain with the boundary data used as an approximation of the solution in the remaining part of the domain. Our result gives a theoretical validation for the use of a non monotone finite element method for the Monge-Ampère equation.

Approximation of Functions with Values in L-space by Adapted Classical Operators [C-20A]

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In this talk, we show that classical approximation operators such as Bernstein, Schoenberg, Modified Schoenberg Operator, and piecewise linear interpolation can be adapted to functions with values in L-spaces (which are generalizations of set-valued and fuzzy-valued functions). We obtain error estimation of approximation for functions with values in L-spaces by such operators, as well as error estimations for some formulas of approximate integration for such functions. The results are used to develop algorithms for the solution of integral equations involving functions with values in L-spaces.

Detecting Singularities in Piecewise-smooth Functions from Their Fourier Coefficients [M-11B]

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We consider the problem of approximating piecewise-smooth periodic functions using their Fourier data. Several existing methods provide full approximation order in the smooth regions, but give only first order accuracy for the jump locations themselves, irrespective of the smoothness of the function between the jumps. We describe an alternative parametric approach to this problem, where the additional orders of smoothness are utilised to the full extent thereby providing a method with best possible rate also for the positions of the discontinuities. If time allows, we also discuss the closely related problem of estimating the parameters of the corresponding generalised exponential sums from perturbed data.

Linear Barycentric Rational Interpolation with Guaranteed Degree of Exactness in Several Dimensions [C-15A]

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The talk will address the problem of constructing a surface from an equispaced sample of a smooth function of two variables. Our approach is an extension of linear barycentric rational interpolation (LBRI). In recent years, this scheme, introduced in 1988 and improved in 2007 by Floater and Hormann, has turned out to be one of the most efficient infinitely smooth interpolants from equispaced data in one dimension (see R.B. Platte, "Algorithms for recovering smooth functions from equispaced data", preprint). However, there does not seem to exist a straightforward way of generalizing it to two–dimensional non rectangular domains. In our presentation we shall present an attempt to extend to some two-dimensional domains the LBRI with guaranteed degree of precision introduced last year.

Inequalities of Generalized p-elliptic Integrals [C-20A]

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In this talk we establish the two-sided inequalities for the generalized p-elliptic integrals K_p, E_p of the first and the second kind, respectively. As well as, we estimate above and below the perimeter $P = \int_0^{\pi_p/2} \sqrt[p]{1 - r^p \sin_p(t)^p} dt = 4aE_p(r)$ of generalized p-ellipse whose parametric equations are $x = a(1 - \sin_p(t)^p)^{1/p}$ and $y = b \sin_p(t)$ for $0 < t < 2\pi_p$, $1 < p < \infty$, where $\sin_p$ are the eigenfunctions of the so-called one-dimensional p-Laplacian

Data Assimilation in Reduced Modeling [M-6A]

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We consider the problem of optimal recovery of an element u of a Hilbert space $\mathcal{H}$ from measurements of the form $\ell_j(u)$, $j = 1, \ldots, m$, where the ℓ_j are known linear functionals on $\mathcal{H}$. Problems of this type are well studied and usually are carried out under an assumption that u belongs to a prescribed model class, typically a known compact subset of $\mathcal{H}$. Motivated by reduced modeling for solving parametric partial differential equations, we consider another setting where the additional information about u is in the form of how well u can be approximated by a certain known subspace V_n of $\mathcal{H}$ of dimension n, or more generally, in the form of how well u can be approximated by each of a sequence of nested subspaces $V_0 \subset V_1 \cdots \subset V_n$ with each V_k of dimension k. This is a joint work with Albert Cohen, Wolfgang Dahmen, Ronald DeVore, Guergana Petrova, and Przemyslaw Wojtaszczyk.

MSN Numerical Techniques for PDEs [M-11B]

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We present a novel technique for the general purpose solution of linear systems of elliptic partial differential equations with inhomogenous boundary conditions. The solution is represented by a piecewise polynomial and a simple collocation technique is used to discretize the PDE and enforce suitable continuity conditions. However, the total number of discrete equations is chosen to be fewer than the number of unknown coefficients in the polynomial representation. A special minimum Sobolev norm solution is then chosen as the approximate solution. Standard compactness arguments can be used to prove that the approximate solution converges to the true solution. The order of convergence of the method is controlled by the choice of Sobolev norm and very high orders have been achieved in practice. The attendant high condition numbers requires the use of specialized numerical techniques to ensure accuracy. A single piece of code has been used to solve a wide variety of problems from Maxwell's equations to fourth order bi-harmonic problems on complex geometries.

Spatially Distributed Sampling and Reconstruction [M-16B]

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A spatially distributed system contains a large amount of agents with limited sensing, data processing, and communication capabilities. Recent technological advances have opened up possibilities to deploy spatially distributed systems for signal sampling and reconstruction. We introduce a graph structure for such distributed sampling and reconstruction systems (DSRS). by coupling agents in a spatially distributed system with innovative positions of signals. We build up a locally verifiable stability criterion for overlapping smaller subsystems. We propose an exponentially convergent distributed algorithm that provides a suboptimal approximation to the original signal in the presence of bounded sampling noises.

Convergence Rates of Derivatives of Floater-Hormann Interpolant [C-15A]

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Floater–Hormann interpolants constitute a family of barycentric rational interpolants which are based on blending local polynomial interpolants of degree d. Recent results suggest that the k-th derivatives of these interpolants converge at the rate of $O(h^{d+1-k})$ for $k \leq d$ as the mesh size h converges to zero. So far, this convergence rate has been proven for $k = 1, 2$ and for $k \geq 3$ under the assumption of equidistant or quasi-equidistant interpolation nodes. In this paper we extend these results and prove that Floater–Hormann interpolants and their derivatives converge at the rate of $O(h_j^{d+1-k})$, where h_j is the local mesh size, for any $k \geq 0$ and any set of well-spaced nodes.

Optimal Random Sampling in Weighted Least Squares Methods [M-6A]

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Least squares methods are of common use when one needs to approximate a function based on its noiseless or noisy observation at n scattered points by a simpler function chosen in an m dimensional space with m less than n. Depending on the context, these points may be randomly drawned according to some distribution, or deterministically selected by the user. In this talk, I shall analyze the stability and approximation properties of weighted least squares method, in relation with the spatial distribution of a random sampling and try to extract the concept of an optimal distribution.

Active Subspaces for Dimension Reduction in Parameter Studies [M-13A]

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Scientists and engineers use computer simulations to study relationships between a physical model's input parameters and its outputs. However, thorough parameter studies—e.g., constructing response surfaces, optimizing, or averaging—are challenging, if not impossible, when the simulation is expensive and the model has several inputs. To enable studies in these instances, the engineer may attempt to reduce the dimension of the model's input parameter space. Active subspaces are a set of dimension reduction tools that identify important directions in the parameter space. I will describe methods for discovering a model's active subspace and propose strategies for exploiting the reduced dimension to enable otherwise infeasible parameter studies. For more info, see www.activesubspaces.org

Asymptotics of the Christoffel-Darboux Kernel for Generalized Jacobi Measures [M-18A]

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Let μ be a finite Borel measure supported on the complex plane and let $p_n(\mu, z)$ be the n-th orthonormal polynomial with respect to μ. The goal of this talk is to study the asymptotic behavior of the Christoffel-Darboux kernel

$$K_n(\mu, z, w) = \sum_{k=0}^{n} \overline{p_k(\mu, z)} p_k(\mu, w)$$

around points where the measure exhibits a power type singularity, for example it is supported on the real line and for some x_0 in its support, μ is absolutely continuous in a neighbourhood U of x_0 with

$$d\mu(x) = w(x)|x - x_0|^\alpha dx, \quad x \in U$$

there for some $\alpha > -1$ and for some positive and continuous weight $w(x)$. We shall establish universality limits for measures supported on the real line

$$\lim_{n\to\infty} \frac{K_n(\mu, x_0 + a/n, x_0 + b/n)}{K_n(\mu, x_0, x_0)},$$

at the singularity x_0 and limits of the type

$$\lim_{n\to\infty} \frac{1}{n^{\alpha+1}} K_n(\nu, z_0, z_0),$$

where ν is supported on a system of Jordan arcs and curves Γ behaving like $d\nu(z) = w(z)|z - z_0|^\alpha ds_\Gamma(z)$ and s_Γ denotes the arc length measure with respect to Γ. Part of the results were established jointly with Vilmos Totik.

Error Bounds for Polynomial Finite Elements on Domains Enclosed by Piecewise Conics [M-10A]

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Error bounds for the finite element method on bivariate domains with boundary defined by a piecewise conic curve will be presented for piecewise polynomial C^0 elements of any degree and C^1 quintic elements. In contrast to the standard curved finite elements and NURBS-based isogeometric finite elements, our construction does not rely on nonlinear geometry mappings, which facilitates C^1 smoothness and applications to fully nonlinear elliptic equations.

New Results in the Theory of Univariate Expo-rational B-splines [C-5B]

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Expo-rational B-splines (ERBS) were introduced by the author in 2002 as special functions which are the asymptotic limits of classical polynomial B-splines when the number of knots of the latter tends to infinity. So far, their theory has been developed only on a relatively basic level, since people from the CAGD community have questioned their usefulness compared to classical polynomial constructions by asking the following three (fairly valid!) questions: 1. why should we use them, if they can only be computed numerically, and not in closed form?; 2. why should we use them, if there is no proof that they achieve the same rates of approximation as the respective polynomial interpolants (concretely, this question has been asked in the context of Hermite interpolation)?; 3. aren't the so-called intrinsic parameters of ERBS-based constructions redundant, and how do we choose their values in a non-arbitrary way? The present talk will provide an exhaustive rigorous answer to these three questions. This will be achieved by the announcing of several important new results in the theory of univariate ERBS, as follows. On question 1: (a) a broad class of ERBS computable in closed form will be added to the recently discovered logistic ERBS (which was the first discovered ERBS to be computable in closed-form); (b) a deep connection will be established between the original default ERBS and the spectral structure of the Fourier transform, resulting in the introduction of a new, Fourier-transform related, ERBS-based, operational calculus which allows the closed-form computation of derivatives and integrals of the original default ERBS of any order (including fractional Riemann-Lioville integrals and Marchaud derivatives), as well as closed-form computation of the Fourier and Laplace transform and the moments of ERBS. On question 2: a new method for estimation of the error remainder in Hermite interpolation will be proposed which is the same for polynomials and for ERBS; based on this approach, it will be shown that, while the error rates are always the same, there are at least six important aspects in which the ERBS interpolant significantly outperforms the respective polynomial interpolant. On question 3: it will be shown that the intrinsic parameters of ERBS and the methods of their optimal selection have important relevance to theoretical approximation theory, operational calculus, functional analysis and operator theory. The present talk can be considered as introductory to the author's talk at the Curves and Surfaces conference in Norway in June, where multivariate ERBS-based constructions will be discussed. The new results relevant to the Fourier transform of ERBS are of key importance for the generation of ERBS-based Calderon functions and the development of a theory of continuous and discrete ERBS-based (multi)wavelet transforms.

Chebyshev Polynomials on a System of Continua [C-10B]

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In 2014, V.V. Andrievskii proved that the uniform norm of the n^{th}-Chebyshev polynomial on a compact set K, consisting of a finite number of quasismooth arcs and Jordan domains with quasismooth boundary such that $\Omega := \mathbb{C} \setminus K$ is a John domain, is bounded by the n^{th} power of the logarithmic capacity of K. In this talk, we use the method of discretizing the equilibrium measure, due to V. Totik, to extend this result to unions of quasiconformal arcs and quasidisks.

A Rescaled Method for RBF Approximation [M-1A]

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In the recent paper (SIAM J. Sci. Comp., 36(6) 2014, A2745–A2762), the authors presented a new method to compute stable kernel-based interpolants termed a rescaled interpolation method. This rescaled interpolation method combines the standard kernel interpolation with a properly defined rescaling operation, which smooths the oscillations of the interpolant. Although promising, this procedure lacks a systematic theoretical investigation. Through our analysis, this novel method can be understood as standard kernel interpolation by means of a properly rescaled kernel. This point of view allow us to consider its error and stability properties. First, we prove that the method is an instance of the Shepard's method, when certain weight functions are used. In particular, the method can reproduce constant functions. Second, it is possible to define a modified set of cardinal functions strictly related to the ones of the not-rescaled kernel. Through these functions, we define a Lebesgue function for the rescaled interpolation process, and study its maximum - the Lebesgue constant - in different settings. Also, a preliminary theoretical result on the estimation of the interpolation error is presented. As an application, we couple our method with a partition of unity algorithm. This setting seems to be the most promising, and we illustrate its behavior with some experiments.

On the Generalized Approximate
Weak Chebyshev Greedy Algorithm [M-6A]

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The Weak Chebyshev Greedy Algorithm (WCGA) is a generalization of the Orthogonal Greedy Algorithm (also known as Orthogonal Matching Pursuit) for a Banach space setting. We consider the modification of the WCGA in which we are allowed to execute each step of the algorithm not precisely but with some inaccuracies in form of relative or absolute errors. Such permission is natural for the numerical applications and simplifies realization of the algorithm. We call this modification the generalized Approximate Weak Chebyshev Algorithm (gAWCGA). We obtain conditions that are sufficient for the convergence of the gAWCGA for an element of a uniformly smooth Banach space, and show that they are necessary in some cases. In particular, we show that if all the errors are from ℓ_1 space then the conditions for the convergence of the gAWCGA are the same as for the WCGA.

Dual Tree Complex Wavelet Transform with More Directions [M-3A]

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Directional Wavelet/Framelet transforms have been widely used to deal with image processing problems. Among the popular transforms, the Dual Tree Complex Wavelet Transform (DT-$\mathbb{C}$WT) is an important one, since (1) its filter banks can be designed in 1D, and implemented in simple tensor product structure for 2D images; (2) the filters are compactly supported in time domain, so the transform can be realized rapidly by time domain convolution and up/downsampling without using FFT. However, the traditional DT-$\mathbb{C}$WT only has 6 directions, which is not enough in real applications. In this talk, we split the original DT-$\mathbb{C}$WT filter bank into a larger one, generating frames with 22 directions in total. Numerical experiments on image denoising problems show our improvements to the original transform.

Quasi-Monte Carlo Methods and PDEs with Random Coefficients [P-7]

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Quasi-Monte Carlo (QMC) methods are equal weight quadrature rules to approximate integrals over the s dimensional unit cube. Classical QMC methods are based on uniformly distributed quadrature points. These QMC rules achieve a convergence rate of almost N^{-1} for integrands of bounded variation, where N is the number of quadrature points. The extension of these methods to higher order QMC rules achieve the optimal rate of convergence also for integrands of smoothness greater than one. QMC and higher order QMC have recently been applied in approximating the expected value of solutions to partial differential equations with random coefficients. In these problems the integrands are infinitely smooth but also infinite dimensional. The latter poses its own set of problems in approximating such integrands. Certain types of higher order QMC rules can be used to overcome these problems. In this talk we give an introduction to QMC and higher order QMC and discuss some of the recent applications in PDEs with random coefficients.

Generating Functions Method for Classical Positive Operators Their q-Analogues and Generalizations [C-5B]

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We present generating functions approach to obtain convergence results for q-analogues of classical positive Bernstein and Baskakov operators and their generalizations.

Splines on Tetrahedral Decompositions [M-1B]

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It is known that the dimension of the space of splines of degree at most d on a tetrahedral decomposition can be given for large d only if dimensions of corresponding bivariate spline spaces are known in all degrees. We make this statement explicit by giving a formula for the dimension of the space of splines of degree d on a tetrahedral decomposition (for large d). The constant term of this formula is the sum of 'excess splines' in all degrees on the cells (vertex stars) of the decomposition. The proof relies on the Schenck-Stillman chain complex for splines; in particular we use a description of the associated primes of the homologies of this chain complex.

Roots of Random Polynomials with Arbitrary Coefficient [M-16A]

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We prove optimal local universality for roots of random polynomials with arbitrary coefficients of polynomial growth. As an application, we derive, for the first time, sharp estimates for the number of real roots of these polynomials, even when the coefficients are not explicit. Our results also hold for series; in particular, we prove local universality for random hyperbolic series.

A Bi-Fidelity, Low-Rank Approximation Technique for Uncertainty Quantification [M-11A]

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This work introduces a model reduction approach that exploits the low-rank structure of the solution of interest for fast propagation of high-dimensional uncertainties. To construct the low-rank approximation, the method utilizes a low-fidelity model to learn a reduced basis and an interpolation rule that can be used to generate an approximation of the high-fidelity solution. Different aspects of this model reduction approach will be illustrated through its application to non-linear, high-dimensional uncertain problems.

Integration and Function Approximation on Grassmannians [M-11B]

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Recently, numerical integration and function approximation on compact Riemannian manifolds based on eigenfunctions of the Laplace-Beltrami operator have been investigated in the literature. Explicit numerical experiments, however, are usually performed for the sphere. Here,we derive numerically feasible expressions for the approximation schemes, and we present for the first time the associated numerical experiments for the Grassmann manifold. Indeed, our experiments match the corresponding theoretical results in the literature.

Adaptive Isogeometric Approximation of Vector Fields Using Hierarchical B-splines [M-13B]

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In this talk, I will present a methodology for adaptive structure-preserving approximation of vector fields arising in electromagnetics, fluid mechanics, and structural mechanics. Our methodology systemically combines two emerging frameworks in isogeometric analysis, spline forests and isogeometric discrete differential forms. Spline forests provide a multi-level, highly adaptive, and geometrically flexible basis function environment while isogeometric discrete differential forms provide a natural means for structure-preserving discretization of vector fields. Their union yields a class of hierarchical B-spline spaces which automatically comprise a discrete de Rham complex. I will provide a brief overview on how to construct adaptive isogeometric vector field approximations, demonstrate that such approximations are stable and indeed comprise a discrete de Rham complex, and present numerical results illustrating the promise of this technology in the context of electromagnetics and creeping flow.

Representation of Functions on Data Defined Trees [M-11B]

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Many current problems dealing with big data can be cast efficiently as function approximation on graphs. The information in the graph structure can often be reorganized in the form of a tree; for example, using clustering techniques. We will present a new system of orthogonal functions on weighted trees. The system is local, easily implementable, and allows for scalable approximations without saturation. A novelty of our orthogonal system is that the Fourier projections are uniformly bounded in the supremum norm. We describe in detail a construction of wavelet-like representations and estimate the degree of approximation of functions on the trees.

One-Bit Compressive Sensing of Dictionary-Sparse Signals [M-18B]

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One-bit compressive sensing has extended the scope of sparse recovery by showing that sparse signals can still be accurately reconstructed even when their measurements are subject to drastic quantization. In the extreme scenario, the measurements are binary — only the sign of each linear sample is maintained. Prior results in one-bit compressive sensing relied on the assumption that the signals of interest are sparse in some fixed orthonormal basis. However, in most practical applications, signals are sparse in a (possibly highly overcomplete) dictionary rather than in a basis. In the classical compressive sensing setting, there has been a surge of activity to obtain recovery guarantees under such a generalized sparsity model. Here, we extend the one-bit framework to this important model, thus completing a unified theory.

Flavors of Compressive Sensing [P4]

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About a decade ago, a couple of groundbreaking articles revealed the possibility to faithfully recover high-dimensional signals from some seemingly incomplete information about them. Perhaps more importantly, practical procedures to perform the recovery were also provided. These realizations had a tremendous impact in science and engineering. They gave rise to a field called "Compressive Sensing", which is now in a mature state and whose foundations rely on an elegant mathematical theory. This talk presents a (biased) survey of the field, accentuating elements of Approximation Theory, of course, but also highlighting connections with other disciplines that have enriched the theory, e.g. sampling theory, geometry of Banach spaces, optimization, frame theory, probability, graph theory, and metagenomics.

Vector Field Approximation Using a Partition of Unity and Customized Radial Basis Functions [M-1A]

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In this talk we introduce an approximation method that employs a partition of unity technique to construct divergence-free or curl-free vector field approximants. After a given domain is decomposed into overlapping patches, local interpolants are constructed on each patch using divergence-free (or curl-free) radial basis functions. The local approximations are then combined in such a way to yield a smooth global approximation that is analytically divergence-free (or curl-free). The method can be used to construct a curl-free approximation in any dimension, while the divergence-free version is currently limited to two-dimensional domains. After introducing the method we provide some preliminary error estimates and several numerical examples.

Function-train: a Continuous Analogue to the Tensor-train Decomposition [M-11A]

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In this talk, we present a method for approximating high-dimensional low-rank functions. Taking advantage of low-rank structure in approximation problems has been shown to prove advantageous for scaling numerical algorithms and computation to higher dimensions by mitigating the curse-of-dimensionality. The method we describe is an extension of the tensor-train cross approximation algorithm to the continuous case of multivariate functions. We begin by describing a new cross-approximation algorithm for computing the CUR/skeleton decomposition of bivariate functions. We then extend this technique to the multidimensional case of the function-train decomposition. Computing the decomposition relies on continuous analogues of matrix factorizations, such as continuous QR and LU factorizations of matrix-valued functions. We finish by showing applications to several areas of interest in uncertainty quantification such as Gaussian filtering and the solution of stochastic elliptic partial differential equations.

Outlier Removal in Data Approximation [C-15A]

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Outliers in measurement data are a major source of approximation inaccuracies. To eliminate outliers, approximation is achieved by a weighted sum of local polynomials and the parameters of the polynomials are determined by a robust estimator. Taking advantage of constraints holding within local data, outliers are removed. The proposed approximation is tested in the context of image registration, determining a transformation function that relates the geometries of two images. Experimental results demonstrate improved registration accuracy by the proposed approximation over thin-plate spline and moving least squares.

Solution Approximation for Inverse Sturm Liouville Problem [C-15A]

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Inverse Sturm Liouville Problem (SLP) aims at recovering the coefficient function of the SLP through the known eigenvalues or eigenfunctions. Practically, only finite number of eigenvalues or eigenfunctions can be provided in constructing such recovery process. The solution thus can only be produced in an approximate way. The approximation includes Fourier expansion modes corresponding to the eigenvalue orders. Several algorithmic techniques including interpolation are herein demonstrated for such approximation. Mathematical and numerical analysis are provided for the schemes. We can thus understand more deeply how the solution approximation procedures should be formed for generating more accurate solutions with high efficiency.

Gradient-enhanced L_1-minimization for Stochastic Collocation Approximations [M-13A]

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This work is concerned with stochastic collocation methods via gradient-enhanced L_1-minimization, where the derivative information is used to identify the Polynomial Chaos expansion coefficients. With an appropriate preconditioned matrix and normalization, we show the inclusion of derivative information will lead to a successful solution recovery, both for bounded and unbounded domains.

Numerical examples are provided to compare the computational performance between standard L_1-minimization and the gradient-enhanced L_1-minimization. Numerical results suggest that including derivative information can accelerate the recovery of the PCE coefficients.

Sampling on Lattices via Radial Basis Functions [M-18B]

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We consider analogues of classical sampling theory for general lattices in $\mathbb{R}^n$ which use interpolation methods involving radial basis functions. The main attentions will be on recovery of bandlimited signals via a limiting process and approximation orders with respect to the lattice spacing. Connections to shift-invariant spaces will be discussed as well as how the interpolation schemes perform in the presence of noise.

Local Approximation with Kernels [M-6A]

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This talk treats the problem of kernel approximation using non-uniform centers. A basic problem is as follows: a global approximant is sought with error which is small in regions where centers are tightly spaced. We present and discuss kernel approximation schemes that deliver these results – specifically, that function without a requirement of quasi-uniformity (a common assumption for kernel approximation), but rather by assuming centers have a locally controlled mesh ratio. Under this type of assumption, and with the aid of recently developed localized bases for RBF approximation, centers may cluster or form gaps without giving rise to instability. At the same time, approximation error is controlled by a local density parameter, so that error responds to the local distribution of data.

Inferring Interaction Rules from Observation of Evolutive Systems [C-20B]

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The description of social dynamics and self-organizing systems via systems of ODEs has attracted a lot of atention in recent years, in particular the seminal work of Cucker and Smale. Here we study a related inverse problem: From observations of such dynamical systems we wish to reconstruct the underlying interaction rules. In particular, for dynamical systems which are obtained from a gradient descent of some energy functional depending only on the mutual distances, i.e. systems of the form

$$\dot{x}_i = \sum_{j \neq i} a(|x_i - x_j|) \frac{x_i - x_j}{|x_i - x_j|}$$

we wish to infer the interaction kernel function a. We present some first results of this ongoing project.

Multivariate Wavelet Frames through Constant Matrix Completion via the Duality Principle [M-8B]

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The duality principle, ultimately a statement about adjoint operators, is a universal principle in frame theory. We take a broad perspective on the duality principle and discuss how the mixed unitary extension principle for MRA-wavelet frames can be viewed as the duality principle in a sequence space. This leads us to a construction scheme for dual MRA-wavelet frames which is strikingly simple in the sense that it only requires the completion of an invertible constant matrix. Under minimal conditions on the multiresolution analysis our construction guarantees the existence and easy constructability of multivariate non-separable dual MRA-wavelet frames of compactly supported wavelets.

Numerical Study of Space-Time RBF for PDEs [M-8A]

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We are experimenting with space-time radial basis function methods for the numerical solutions of time-dependent PDEs. Unlike common numerical schemes where space and time are treated independently (known as the method of lines), the time variable in space-time formulation is treated as another space variable. The discretized version of PDEs are then solved as boundary value problems in space-time domain. Several numerical examples will be presented.

Gaussian Quadrature Rules for Splines and their Application in Isogeometric Analysis [M-15B]

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We continue previous work in search of Gaussian quadrature rules for univariate splines. Such rules are optimal in the sense that there exists no other quadrature rule that can exactly integrate all elements of the function space with less quadrature points. Although existence and uniqueness properties have not been established in the general setting, we are able to numerically compute Gaussian quadrature rules for a range of spline spaces of practical interest. We provide some of these rules in a useful format for the user and show their efficacy applied to two- and three-dimensional diffusion-reaction problems in the context of exact and reduced quadrature.

Numerical Stability of Best Approximations in a Frame [M-6B]

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A frame is a redundant set, and for this reasons a simple approximation technique like interpolation exhibits strong ill-conditioning: there are multiple answers. In this talk we analyze the numerical stability of frame approximations and show that, with some care, the results are actually very positive. The most stable technique is a discrete least squares approximation, in spite of extreme ill-conditioning of the normal equations. Such approximations differ from the standard definition in terms of the canonical dual, and we show some examples where rapid convergence rates are obtained to very high accuracy.

Efficient Sampling Schemes for Recovering Sparse Orthogonal Polynomials [M-11A]

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ℓ_1-minimization is an efficient technique for estimating the coefficients of a Polynomial Chaos Expansion (PCE) from a limited number of model simulations. In this talk I will present a ℓ_1 approximation scheme that leverages the relationship between the Christoffel function and equilibrium sampling to accurately approximate high-dimensional models using a large class of orthonormal polynomials. The efficacy of this method will be demonstrated with various theoretical and numerical results.

Quark Frames in Adaptive Numerical Schemes for the Solution of PDEs [M-8A]

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A quarkonial decomposition of a function space is based on a partition of unity (PUM) whose elements are not only translated and dilated as in the wavelet case, but also multiplied by polynomials up to a certain order. By proceeding this way , the collection of atoms is highly enriched and therefore allows for much more flexible decomposition strategies. However, on the other hand, the representation of a given function then gets highly redundant. Therefore, we do not end up with a basis, but with a frame. Once we end up with a Quark frame it is clearly an interesting and challenging task to design tailored adaptive numerical schemes. In the long run, by combining the knowledge on the design of adaptive wavelet methods with the concept of quarkonial decompositions, it might be possible to derive very powerful schemes with a provable order of convergence

Polynomial Approximation on a Sparse Grid [C-20A]

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We study polynomial approximation on a certain class of sparse grids that we call quasi-uniform. Using Boolean sum techniques combined with a Bernstein class of dual bases in subspaces, we construct quasi interpolants on these grids. Our construction achieves rates of approximation analogous to those on a tensor product grid, while retaining some geometric properties of Bernstein-Bézier surfaces. See S. Kersey, *Discrete Polynomial Blending*, arXiv: 1602.08365 and S. Kersey, *Dual Basis Functions in Subspaces*, arXiv: 1406.6632.

Approximation of the Borel-Tanner Distribution [C-10B]

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The Borel-Tanner distribution also known as the generalized Poisson distribution is a three parameter distribution. This distribution is becoming increasingly useful in many branches of sciences specially related to single-server steady state queueing processes. Generalized distributions are becoming increasingly evident and useful in many branches of science but the functional forms of these generalized distributions are often complicated. Therefore, there arises a need to have some simplified or approximated form of this generalized distribution and also to know their relations with other distributions. Here we approximate the Borel-Tanner distribution by using different techniques and suggested the best approximation. We also derive the standard normal approximation of generalized Poisson distribution. The results are intended to fill a conspicuous gap in the mathematical and statistical literature concerning the empirical quality of the approximations, and they are useful for designing efficient and accurate computing algorithms for such probabilities.

Flux Conservative Hermite Methods for Nonlinear Conservation Laws [C-10A]

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Numerical methods for hyperbolic conservation laws must be high order accurate for smooth solutions and have shock capturing capability for non-smooth solutions. In this talk, we present a new class of methods built on polynomial approximation and Hermite interpolation that we use to approximate solutions to nonlinear conservation laws. We show how the Entropy Viscosity method, which constructs a local artificial nonlinear viscosity based on the entropy residual, can be adopted to Hermite framework. In theory, the Entropy Viscosity method is supposed to add dissipation only at shocks, but in practice, after numerical discretization, it also adds artificial viscosity along contact discontinuities. In this talk, we also describe a simple mechanism that removes this deficiency.

MRA-based Wavelet Frames with Symmetry [M-8B]

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Construction of multivariate symmetric wavelets is important for various engineering applications. For an arbitrary symmetry group H, we give explicit formulas for H-symmetric refinable masks with sum rule of order n. The description of all such masks is given. We suggest several methods for the construction of H-symmetric wavelets in different setups (frame-like systems and frames) providing approximation order n. Some similar results concerning a multi-wavelet case are also considered.

A New Flat Extension Theorem [C-13A]

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A generalization of the the flat extension theorems of Curto and Fialkow and Laurent and Mourrain is obtained by using techniques of ideal projections. Along the way, it is seen that the moment matrix of a linear functional on $\Pi(\mathbb{C}^k)$ has finite rank iff the functional is finitely atomic, a fact previously known for positive functionals.

Fuglede Problems in Gabor and Wavelet Analysis [M-3B]

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Fuglede conjecture asserted that a set (called spectral set) Ω admits an exponential orthonormal basis in $L^2(\Omega)$ if and only if Ω is a translational tile. It was disproved in high dimension by Tao in 2004. However, spectral sets and tiles are actually fundamental building block of Gabor bases and wavelet sets. In this talk, we will give a report on our recent investigation on the structure of Gabor orthonormal bases and wavelet sets from the point of view of the Fuglede problems.

Spline Solution of Second Order Elliptic PDE in Nondivergence Form [M-3B]

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I will explain how to use bivariate splines to solve 2nd order elliptic PDE in non-divergence form. We formulate the solution as a minimization problem and first show the existence and uniqueness of spline solution. Then we present numerical results. Comparison with existing results will be presented.

A Polygonal Spline Method for General 2nd-Order Elliptic Equations and its Applications [M-3A]

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As recently as last year, Floater & Lai extended bivariate spline methods for numerical solution of PDEs to a new method involving polygonal splines, which are well-defined over polygonal partitions rather than merely triangulations. While this method is more computationally expensive, it was shown to provide more accurate solutions with less degrees of freedom compared to the traditional polynomial finite element method, at least when solutions are smooth. In their paper, they examined numerical solutions of Poisson equations using this method. In this paper, we have extended their results to general second-order elliptic PDEs; that is, we have modified the method to numerically solve PDEs of the form $-\sum_{1\le i,j\le n} \frac{\partial}{\partial x_j}\left(A_{ij}\frac{\partial u}{\partial x_i}\right) + \sum_{k=1}^{n} B_k \frac{\partial u}{\partial x_k} + Cu = f$ with Dirichlet boundary condition, with $A_{ij} \in L_\infty(\Omega)$ with the matrix $[A_{ij}]_{1\le i,j\le n}$ positive definite, $B_k \in L_\infty(\Omega), C \in L_\infty(\Omega)$, and f a function in $L_2(\Omega)$. We apply the method directly to some elliptic PDEs to show some convergence results, along with some singularly perturbed parabolic and hyperbolic PDEs as a new application.

Meshfree Approximation of PDEs: Recent Developments in Radial Basis Function Approximation [P-17]

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When solving partial differential equations with smooth solutions it is often beneficial from a computational cost perspective to use high order approximation methods. For specific regular geometries highly efficient pseudo-spectral methods are available. For more irregular geometries, meshfree radial basis function (RBF) approximation provides an easy way to construct high order approximations. However, when global RBFs are employed, dense linear systems need to be solved, and the computational cost becomes too high both for large computational domains and for high-dimensional problems. Currently, the research field is moving towards localized RBF approximation, where the main directions are stencil-based approximations (RBF-FD) and partition of unity approximations (RBF-PUM). For the localized approaches, the involved linear systems become sparse, and significant savings in computational and storage costs are achieved. By examining what RBF approximation theory can tell us, we find the clues for how to construct an RBF-PUM method that is both computationally efficient and numerically stable for large scale problems. We show numerical results for a series of increasingly stable formulations in two space dimensions.

Refinable Functions with PV Dilations [M-8B]

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A PV number is an algebraic integer α of degree $d \geq 2$ all of whose Galois conjugates other than itself have modulus less than 1. Erdös proved that the Fourier transform, of any nonzero compactly supported function satisfying the refinement equation $\varphi(x) = \frac{\alpha}{2}\varphi(\alpha x) + \frac{\alpha}{2}\varphi(\alpha x - 1)$ with PV dilation α, does not vanish at infinity so by the Riemann-Lebesgue lemma φ is not integrable. Dai, Feng and Wang extended his result to solutions of $\varphi(x) = \sum_k a(k)\varphi(\alpha x - \tau(k))$ where $\tau(k)$ are integers and a has finite support and sums to $|\alpha|$. In 2014 we conjectured that their result holds under the weaker assumption that τ has values in the ring of polynomials in α with integer coefficients. This paper formulates a stronger conjecture and provides support for it based on deep results of Erdös and Mahler, James, and others that characterize the asymptotic density of integers represented by integral binary forms. We show that our conjecture fails for vector valued refinable functions by constructing a simple counterexample

On a Connection between Nonstationary and Periodic Wavelets [M-8B]

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We study a connection between nonstationary nonperiodic wavelets and periodic wavelets. The case of Parseval wavelet frames generated by the unitary extension principle is discussed. It is proved that the periodization of a nonstationary Parseval wavelet frame is a periodic Parseval wavelet frame. And conversely, a periodic Parseval wavelet frame generates a nonstationary wavelet system. There are infinitely many nonstationary systems corresponding to the same periodic wavelet. Under natural conditions on periodic scaling functions, among these nonstationary wavelet systems there exists a system such that its time-frequency localization is adjusted with an angular-frequency localization of an initial periodic wavelet system. Namely, we get the following equality $\lim_{j\to\infty}(UC_B(\psi_j) - UC_H(\psi_j^0)) = 0$, where UC_B and UC_H are the Breitenberger and the Heisenberg uncertainty constants, $\psi_j \in L_2(\mathbb{T})$ and $\psi_j^0 \in L_2(\mathbb{R})$ are periodic and nonstationary wavelet functions respectively.

Polyhyperbolic Cardinal Splines [M-16B]

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In this talk we discuss solutions of differential equation $(D^2 - \alpha^2)^k u = 0$ on $\mathbb{R} \setminus \mathbb{Z}$, which we call hyperbolic splines. We develop the fundamental function of interpolation and discuss various properties related to these splines.

Explicit Formulas for the Distribution of Complex Zeros of Random Polynomials [M-16A]

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Inspired by the works of Shepp and Vanderbei on random polynomials, we present explicit formulas for the distribution of complex zeros of random polynomials of the form $\sum_{j=0}^{n} \eta_j f_j(z)$, where z is a complex variable, the coefficients η_j are independent complex standard normal random variables, and the functions f_j are given analytic functions that are real-valued on the real number line. We consider, also, some special cases such as random Weyl polynomials and random truncated Fourier sine and cosine series.

Adaptive Algorithms using Splines on Triangulations with Hanging Vertices [C-5B]

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Adaptive approximation of functions is tested using polynomial splines on triangulations with hanging vertices and indicates improved efficiency as compared to ordinary triangulations. Algorithms for generating data structures needed for triangulations with hanging vertices are also developed. Adaptive mesh generation algorithms using the finite element method(FEM) for solving a model problem involving a second order elliptic PDE are also discussed. Numerical examples using different *a posteriori* error indicators are given.

Sparse Recovery in l_p $(0 < p < 1)$ with Dictionary [M-18B]

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In this talk, i shall investigate sparse recovery in l_p $(0 < p < 1)$ with dictionary. By introducing generalized D-RIP, we characterize sparse signal recovery problem under dictionary.

Scaling Limits of Polynomials and Entire Functions of Exponential Type [P-14]

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Scaling limits play an important role in many areas of analysis. One of the simplest is one we teach in elementary calculus, namely that $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$. On the left-hand side, we have a sequence of polynomials with the scaled variable x/n. On the right-hand side, we have an entire function of exponential type. We discuss some topics where this type of limit plays a role, and where there are still unsolved problems: **(I) The Bernstein Constant of Polynomial Approximation** Or, what is the distance from $|x|$ to polynomials of degree at most n? **(II) Universality Limits in Random Matrix Theory** Or, what do eigenvalues of random Hermitian matrices have to do with orthogonal polynomials and their reproducing kernels? **(III) Nikolskii Inequalities** How do the size of polynomials shape up in different L_p and L_q norms? **(IV) Marcinkiewicz-Zygmund and Polya-Plancherel Inequalities** What do values of polynomials at equally spaced points in the circle have to do with values of entire functions at the integers?

The Geometry of Random Lemniscates [M-18A]

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A (rational) lemniscate is the level set of the modulus of a rational function. While sampling from an ensemble of random lemniscates that is invariant under rotations of the Riemann sphere, we study basic geometric and topological properties. For instance, what is the average spherical length of a random lemniscate? How many connected components are there and how are they arranged in the plane?

Dimension of Tchebycheffian Spline Spaces on T-meshes [M-1B]

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The dimension of polynomial spline spaces on a prescribed T-mesh for a given component-wise degree and smoothness has been addressed by several authors using different techniques. In this talk we consider the dimension problem for general Tchebycheffian splines using a homological approach. The extension is non-trivial because the ring structure of algebraic polynomials cannot be used in this general setting. The results strengthens the structural similarity between algebraic polynomial and general Tchebycheffian spline spaces

Generalized B-splines in Isogeometric Analysis [P-12]

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Generalized splines are smooth piecewise functions with sections in spaces more general than classical algebraic polynomials. Interesting examples are spaces comprising trigonometric or hyperbolic functions. Under suitable assumptions, generalized splines enjoy all the desirable properties of polynomial splines, including a representation in terms of basis functions (the so-called GB-splines) that are a natural extension of the polynomial B-splines. Isogeometric analysis (IgA) is a well-established paradigm for the analysis of problems governed by partial differential equations. It provides a design-through-analysis connection by exploiting a common representation model. This connection is achieved by using the functions adopted in Computer Aided Design (CAD) systems not only to describe the domain geometry, but also to represent the numerical solution of the differential problem. CAD software, used in industry for geometric modelling, typically describes physical domains by means of tensor-product B-splines and their rational extension, the so-called

NURBS. In its original formulation IgA is based on the same set of functions. Nonetheless, the IgA paradigm is not confined to B-splines, NURBS and their localized extensions. Thanks to their complete structural similarity with classical B-splines, GB-splines are plug-to-plug compatible with B-splines in IgA. On the other hand, when dealing with GB-splines, the section spaces can be selected according to a problem-oriented strategy taking into account the geometrical and/or analytical peculiar issues of the specific addressed problem. The fine-tuning of the approximation spaces results in a gain from the accuracy point of view. In this talk we review some isogeometric methods based on trigonometric and exponential generalized spline spaces for their relevance in practical applications, discussing the differences and the similarities with the polynomial case.

Sensor Placement for Inferring Data-driven Dynamics [C-18B]

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Point sensors or measurements can offer better insight into structural dynamics of data for most physical phenomena with broad correlated structures. A dimension reduced approximation to data is obtained using Principal Component Analysis (PCA). Point sensor locations are chosen that capture directions of maximal variation in PCA feature space. These selected sensors demonstrate improved performance over random point sensors for tasks as diverse as categorical classification and future state prediction of data, and are more robust to sensor noise. We demonstrate the advantages of this sensor placement algorithm over random measurements for two applications. The first is a classification of aerodynamic environments from limited measurements of wing strain in flapping wing flight using compressed sensing. The second involves predicting sizes of shims (filler components) between two mating parts in aircraft assembly given limited measurements of the gaps between them.

Fourier Frames: Algorithms and Applications [M-6B]

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Fourier Series are a very useful tool in function approximation thanks to their fast transform, easy differentiability and spectral convergence under the right conditions. Unfortunately these conditions are restrictive, requiring the approximant to be periodic in all dimensions. In this talk we present algorithms to approximate non-periodic functions on equispaced grids using Fourier Frames. These consist of Fourier basis functions periodic on a bounding box, but restricted to an arbitrary subdomain. The resulting redundant set has very nice approximation capabilities, but the approximations can no longer be obtained quickly through the FFT. Algorithms also need to account for extreme ill-conditioning of the Gramian and collocation matrices because of the redundancy. We present an algorithm that exploits this ill-conditioning to compute an approximation orders of magnitude faster than through inverting the full matrix. As an example of the versatility of this algorithm, we easily adapt it to solve constant coefficient differential equations on complicated geometries.

A Moving Mesh Method for Numerical Solution of PDEs [M-3B]

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We study a globally adaptive mesh for numerically solving PDEs through the use of bivariate splines. Based on a previous solution, we estimate the location of sharp changes and flat spots; then, we add vertices to and subtract vertices from the existing mesh to generate a new one. Since the new triangulation is generated by the Delaunay method, it offers several advantages over locally adjusted moving mesh methods. Together with the flexibility of the spline method for arbitrary degree and smoothness of solutions, our approach provides a more effective way to solve PDEs numerically.

Adaptive and Anisotropic Approximation Tools and Techniques

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We study approximation procedures featuring both adaptivity and anisotropy, which makes them strongly non-linear and brings up several unexpected mathematical structures. In the first part of the talk, we consider finite element approximation on adaptive and anisotropic unstructured meshes, in Sobolev norm. We show that the optimal rates of convergence are governed by Hilbert's invariant polynomials. The structure of asymptotically optimal meshes combines high anisotropy and in some cases a maximal angle condition. This combination of geometric constraints rigidifies the mesh in a manner that can be reformulated analytically, using the framework of mesh-metric equivalence. In the second part of the talk, we study approximation under the constraint of convexity, with a cartesian grid as the imposed point set. Anisotropy again plays a key role, but the different constraints require the use of different mathematical tools. The key concepts here are reduced bases of additive lattices, and the Stern-Brocot arithmetic tree. These objects, rather new to the field of numerical analysis, turn out to be versatile and useful in the discretization of numerous anisotropic PDEs, ranging from Monge-Ampere to Eikonal equations.

B-spline Finite Elements on Smooth Surfaces [M-10A]

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Finite Elements are a standard method for solving PDEs. We are interested in PDEs defined on smooth surfaces. In this talk we show how to construct Finite Element spaces based on B-splines for compact smooth surfaces. The main idea is to define Finite Element-spaces on the charts representing the surface, which are then combined by a partition of unity method. Stability an convergence are the most important criteria for the quality of Finite Element spaces. We show that these new spaces have stable bases and inherit the approximation order from the underlaying splines spaces.

Quasi Optimality for Petrov-Galerkin Solutions of Parabolic Random PDEs in Full Space-time Weak Form [M-8A]

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We consider parabolic evolution problems with random coefficients formulated in a full space and time weak sense. Having strict uniform bounds for the spatial operator, the almost sure existence and uniqueness of a solution is inherited from the deterministic case by a pathwise treatment. Now we relax this restriction of having uniform bounds and allow them to be random variables instead. That means, that the elliptic operator does not need to be bounded form above and below by strict constants, but rather by random variables which may depend on the stochastic parameter. We allow the coercivity constant to tend to zero and/or the continuity constant to tend to infinity. Therefore also unbounded operators are covered. Depending on the number of existing moments for these bounds, we can prove existence of p-moments for the solution. Moreover, we go along similar lines and show existence of p-moments for Petrov-Galerkin solutions and also prove quasi optimality in suitable random L_p-spaces. We will be able to show quasi optimality with uniform constant independent of the refinement level of the discretization as well as the stochastic parameter for suitable discrete subspaces.

Dictionary Data Assimilation for Recovery Problems [M-6A]

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We consider the optimal recovery of an element u of a Hilbert space $\mathcal{H}$ from the knowledge of a given finite dimensional subspace V and the knowledge of m measurements. The measurements are given in the form $\ell_i(u),\ i = 1, \ldots, m$, where the ℓ_i are linear functionals of $\mathcal{H}'$. In this work, we discuss one particular question that arises in this context which is the one of the selection of the best m linear functionals when they can be chosen among a dictionary of possible candidates.

A Meshless Kernel-Based Galerkin Method for Non-Local Diffusion Problems [M-1A]

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In this talk, we will discuss a recently introduced meshless Galerkin method for solving both continuous and discrete variational formulations of a volume constrained, nonlocal diffusion problem. Our method is nonconforming and uses a localized Lagrange basis that is constructed out of radial basis functions. By verifying that certain inf-sup conditions hold, we demonstrate that both the continuous and discrete problems are well-posed. We also present numerical and theoretical results for the convergence behavior of the method.

From Patches to RAGS [P-19]

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In this survey talk we will be concerned with rational geometric splines (or RAGS) on general 3D triangulations. Among other things, such splines are suitable for representing smooth parametric surfaces of arbitrary topological genus and arbitrary order of continuity. RAGS are a generalization of the classical bivariate polynomial splines on planar triangulations. The idea of their construction is to utilize linear fractional transformations (or transition maps) to glue together adjacent triangles of the given 3D triangulation. Such transition maps are ideally suited for defining rational splines because the composition of a rational function of type n/n with a linear fractional transformation is again of type n/n. In this talk we will discuss the genesis of the ideas behind the construction of RAGS, describe main results, and also contemplate on some remaining challenges.

Constrained Adaptive Sensing [M-18B]

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Suppose that we wish to estimate a vector from a small number of noisy linear measurements. When the vector is sparse, one can obtain a significantly more accurate estimate by adaptively selecting the samples based on the previous measurements. In this talk we consider the case where we wish to realize the benefits of adaptivity but where the samples are subject to physical constraints. We demonstrate both the limitations and advantages of adaptive sensing in this constrained setting.

Sparse Recovery from Saturated Measurements [M-18B]

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We will discuss a novel signal acquisition framework for compressive sensing. Our goal is to bridge the gap between classical compressive sensing and one-bit compressive sensing. In the latter setup, only the signs of a signal are available and reconstruction is only possible up to the signal's direction. The framework studied here allows for exact signal acquisition up to a specified threshold, after which signals become saturated and only their signs are available. Taking the saturation threshold to zero or infinity, this reduces to the one-bit or classical setup, respectively. One might expect that signals of small magnitude can be recovered exactly from saturated measurements, while signals of large magnitude can only be recovered up to their direction. We will make this statement precise and also show that reconstruction error can be controlled in an intermediate-magnitude regime.

Real Roots of Random Polynomials: Expectation and Repulsion [M-16A]

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Let P be a Kac random polynomial where the coefficients are iid copies of a given random variable x. Under general conditions on x, we give an optimal bound on the probability that P has a double real root. In application we show that the expected number of real roots of P is 2 log n/pi +C +o(1), where C is an absolute constant depending on x. Prior to this result, such a result was known only for the case when x is Gaussian.

Balian–Low–type Theorems for Shift-invariant Spaces [M-16B]

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When shift-invariant spaces and Gabor systems are used as approximation spaces, it is advantageous for the generators of such spaces to be localized and for the spaces to be representative of a large class of functions. However, the celebrated Balian-Low Theorem shows that if a Gabor system generated by a function forms an orthonormal basis for $L^2(\mathbb{R})$, then the function must be poorly localized in either time or frequency. In this talk, I will discuss similar restrictions on the generators of finitely-generated shift-invariant spaces. In particular, I will show that if the integer translates of a well-localized function, $f \in L^2(\mathbb{R}^d)$, form certain types of bases for the shift-invariant space generated by f, then this space cannot be invariant under any non-integer shift.

New Properties of a Certain Method of Summation of Generalized Hypergeometric Series [C-18A]

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In this talk we discuss an approximation of some special functions using a certain method for summation of slowly convergent generalized hypergeometric series. The aim of the talk is to give new theoretical properties of this transformation, including the convergence acceleration theorem. Also, the performance of the method will be depicted by illustrative numerical examples.

Generalized Phase Retrieval: Isometries in Vector Spaces [C-5A]

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In this talk, a generalization of the problem of phase retrieval will be presented. Our problem concerns the minimum number of measurements required to identify a multivector up to some special classes of isometries in the space. We give some upper and lower estimates on the minimal number of multilinear operators needed for the retrieval. The results are preliminary and far from sharp.

Hierarchical Box Splines in Isogeometric Analysis [M-13B]

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In the context of Isogeometric Analysis (IgA) and in the numerical treatment of PDEs in general, efficient local refinement procedures and an easy modeling of complex geometries are crucial ingredients. Splines on regular triangulations equipped with suitable bases, are the natural bivariate generalization of univariate B-splines and can be seen as an intermediate step between tensor product structures and general triangulations. They can be easily extended to higher dimensions, and therefore represent a promising geometric tool in IgA. On the other hand, local refinements can be achieved by considering hierarchically nested sequences of box spline spaces.
In this talk we aim to report about our ongoing work concerning hierarchical bivariate box splines defined on regular three-directional meshes and their use in IgA. The boundary conditions are enforced in a weak form by introducing a special domain strip, whose thickness is adaptively defined. Numerical examples show the optimal convergence of the presented box spline spaces and their hierarchical variants.

Asynchronous Parallel Computing in Signal Processing and Machine Learning [M-20B]

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Finding a fixed point to a nonexpansive operator, i.e., x=Tx, abstracts many problems in numerical linear algebra, optimization, and other areas of scientific computing. To solve fixed-point problems, we propose ARock, an algorithmic framework in which multiple agents (machines, processors, or cores) update x in an asynchronous parallel fashion. Asynchrony is crucial to parallel computing since it reduces synchronization wait, relaxes communication bottleneck, and thus speeds up computing significantly. At each step of ARock, an agent updates a randomly selected coordinate xi based on possibly out-of-date information on x. The agents share x through either global memory or communication. If writing xi is atomic, the agents can read and write x without memory locks. Theoretically, we show that if the nonexpansive operator T has a fixed point, then with probability one, ARock generates a sequence that converges to a fixed points of T. Our conditions on T and step sizes are weaker than comparable work. Linear convergence is also obtained. We propose special cases of ARock for linear systems, convex optimization, machine learning, as well as distributed and decentralized consensus problems. Numerical experiments of solving sparse logistic regression problems are presented.

Frames and Bessel Systems of Vectors Generated by Actions of Operators [M-16B]

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We consider systems of vectors of a form

$$\{A^n g_i :\ i \in I, n \geq 0\}$$

where $\{g_i\}_{i\in I}$ is a countable (finite or infinite) subset of a separable Hilbert space $\mathcal{H}$ and $A \in B(H)$ is a normal operator. We show that a system of that form can never be complete and minimal and find conditions that the operator A needs to satisfy for the system to be complete and Bessel. We also investigate systems of a form

$$\{\pi(\gamma) g_i :\ i \in I, \gamma \in \Gamma\}$$

where π is a unitary representation on $\mathcal{H}$ of the discrete group Γ and extract information about the spectrum of the operators in the group when the system is minimal or complete and Bessel.

A Wavelet Collocation Method for a Fractional Differential Problem [C-5A]

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We introduce a multiscale collocation method to numerically solve differential problems involving ordinary and fractional derivatives of high order. The proposed method uses multiresolution analyses (MRA) as approximating spaces and takes advantage of a finite difference formula that allow us to express both ordinary and fractional derivatives of the approximating function in a closed form. Thus, the method is easy to implement, accurate and efficient. The convergence and the stability of the multiscale collocation method are proved and some numerical results are displayed.

Orthogonal Polynomials on the Unit Ball with a Mass on the Sphere [C-16A]

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We study a family of mutually orthogonal polynomials on the unit ball with respect to an inner product which includes a mass uniformly distributed on the sphere. First, connection formulas relating these multivariate orthogonal polynomials and the classical ball polynomials are obtained. Then, using the representation formula for these polynomials in terms of spherical harmonics, differential and analytic properties will be deduced.

A Fast Radial Basis Functions Method for Solving PDEs on Arbitrary Surfaces [M-6B]

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The RBF Orthogonal Gradients Method (OGr) allows us to compute differential operators restricted to general surfaces in $\mathbb{R}^3$ just by means of point clouds. It also benefits from RBFs' strengths: simplicity, high accuracy and also a meshfree character, which gives the flexibility to represent the most complex geometries. We are introducing a fast version of the OGr algorithm, which makes use of RBF-generated finite differences to discretize the differential operators.

Distribution of Zeros for Random Laurent Polynomials [M-18A]

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We study the global distribution of zeros for random Laurent polynomials. The zero counting measures of such polynomials converge weakly (with probability one) to a linear combination of uniform distributions on two concentric circles under certain natural assumptions on the coefficients. This holds for i.i.d. coefficients with finite $\log^+$ expectations, and for non-i.i.d. coefficients with uniform bounds on tails of distributions. We also consider applications to the zeros of associated random trigonometric polynomials, and quantify convergence of the zero counting measures via expected discrepancy.

Zero Distribution of Random Orthogonal Polynomials [M-16A]

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Let $G \subset \mathbb{C}$ be a bounded simply connected domain with analytic boundary $\partial G = E$. Consider a sequence of random polynomials $P_n(z) = \sum_{k=0}^{n} A_k B_k(z)$, where the A_k are i.i.d random variables and the B_k are a nice basis of orthogonal polynomials (say Bergman orthogonal). Let τ_n denote the normalized counting measure of the zeros of P_n. We show that $\tau_n \to \mu_E$ iff $\mathbb{E}(\log^+ |A_0|) < \infty$, where μ_E denotes the equilibrium measure of E.

Approximation and Modeling with Ambient B-Splines [P-2]

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The approximation of data given on some manifold is a difficult task since, in general, it is not clear how to construct finite-dimensional function spaces with favorable properties on that manifold. A natural idea is to extend the given data to the ambient space of the manifold, for instance by requesting constant values along lines perpendicular to the manifold. In this way, a new function is defined, which can be approximated by standard tensor product splines in the ambient space.

Restricting these splines to the manifold yields the desired approximation. We show that this method has optimal approximation power and illustrate its properties by practical examples. The method can also be used to define parametrizations of free-form surfaces of arbitrary topology. Unlike other approaches, like geometric continuity or subdivision, the ambient B-spline method yields any desired order of smoothness easily. In the second part of the talk, we discuss the potential of the method for the solution of intrinsic partial differential equation on manifolds. Here, not a given function but functionals like the Laplace-Beltrami operator must be extended to the ambient space of the manifold. Once this is done, the resulting higher-dimensional PDE can be approximated by tensor product B-splines, and again, the actual solution is found by restricting the solution to the manifold. Theorems on the existence and uniqueness of solutions are provided, while convergence results are not available, yet.

Entire Functions of Exponential Type and the Bernstein Constant [C-20A]

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Let $\alpha > 0$ be not an even integer. In 1913 and later in 1938, S.N. Bernstein established the limit relation $\Delta_{\alpha,\infty} = \lim_{n\to\infty} n^{\alpha} E_n\left(|x|^{\alpha}, L_{\infty}[-1,1]\right)$. Here, $E_n\left(f, L_p[a,b]\right)$ denotes the error in best L_p approximation of a function f on the interval $[a,b]$ by polynomials of maximal degree $n \in \mathbb{N}$. The constants $\Delta_{\alpha,\infty}$ are the so-called Bernstein constants. There is not a single value of α for which $\Delta_{\alpha,\infty}$ is explicitly known. In this talk we report on asymptotic relations for the approximation of $|x|^{\alpha}, \alpha > 0$ in $L_{\infty}[-1,1]$ by Lagrange interpolation polynomials based on the zeros of the Chebyshev polynomials of first kind. The limiting process reveals an entire function of exponential type for which we can present an explicit formula. We also present a compilation of numerical results involving some non-trivial linear combinations of certain interpolation polynomials, in order to present explicit formulas for near best approximation polynomials in the L_{∞} norm. Possibly and hopefully these formulas could indicate a feasible direction towards some explicit asymptotic representations of best approximation polynomials for $|x|^{\alpha}$ in the L_{∞} norm and thus for the Bernstein constants $\Delta_{\alpha,\infty}$ themselves.

Kernel Based Localization for the Numerical Solution of Parametric PDEs [M-8A]

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In this talk, we will first outline how reproducing kernels naturally arise in the context of parametric partial differential equations. Once a kernel has been chosen, there are many well established numerical methods to solve the given reconstruction problem. A major drawback of many of those methods is that one faces large densely populated ill-conditioned matrices. There are several methods to localize the kernel matrices, i.e., to approximate them by sparser matrices. We will quantify the influence of this matrix approximation error on the final reconstruction error. This is partly based on joint works with M. Griebel (Bonn University), T. Hangelbroek (University of Hawaii), F. Narcowich, J. Ward (both Texas A&M) and P. Zaspel (Heidelberg University).

The Complete Parametrization of the Length Twelve Orthogonal Wavelets [C-5A]

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In this talk, a complete parametrization of the length twelve wavelets is given for the dilation coefficients of the trigonometric polynomials, $m(\omega)$, that satisfy the necessary conditions for orthogonality, that is $m(0) = \sqrt{2}$ and $|m(\omega)|^2 + |m(\omega + \pi)|^2 = 2$. This parametrization has five free parameters and has a simple compatibility with the shorter length parametrizations for some specific choices of the free parameters. These wavelets have varying numbers of vanishing moments and regularity, but continuously transform from one to the other with the perturbation of the free parameters. Finally, we graph some example scaling functions from the parametrization which include the standard wavelets and some new wavelets that perform comparable to the biorthogonal 9/7 wavelet in an image compression experiment on several images.

High Dimensional Density Estimation With Sparse Grids [M-11A]

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Non-parametric density estimation is a well studied problem in modern statistics. In the low dimensional setting there are a number of popular methods which are well understood and efficient. In contrast, high dimensional density estimation remains a challenging problem, with no clear consensus on the best approach. As the dimension of the variable space increases there is an exponential growth in both the computational complexity of the problem and in the required sample size. This is a fundamental difficulty, and no single technique can avoid it altogether. Instead, techniques must be developed that take advantage of problem specific information or simplifying assumptions. With an added assumption of regularity we are allowed a more efficient grid based approximation of a density function, in particular using Sparse Grids. This grid approach allows an efficient treatment of massive data sets, while still possessing a reasonable degree of flexibility. We will present some options for estimating density functions using Sparse Grids. Theoretical properties and numerical examples of real world performance of the proposed methods will be presented.

L^p Norms of Polynomials Orthogonal with respect to Weights w such that $w, w^{-1} \in$ BMO [C-18A]

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Using the iterated commutators of Coifman, Rochberg and Weiss we prove estimates on the $L^p(\mathbb{T})$-norm of polynomials associated to measures which are absolutely continuous with respect to weights w such that $w, w^{-1} \in \mathrm{BMO}(\mathbb{T})$. As an application, this allows us to give asymptotics for polynomial entropies in this case as well.

Spline-based Numerical Schemes for Boundary Integral Equations [M-15B]

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Boundary Element Methods (BEMs) are schemes studied since the mid '80, for the numerical solution of those Boundary Valued Problems, which can be transformed to Boundary Integral Equations. Indeed, if the fundamental solution of the differential operator is known, a wide class of elliptic, parabolic, hyperbolic, interior and exterior problems can be reformulated by integral equations defined on the boundary of the given spatial domain, whose solution is successively obtained by collocation or Galerkin procedures. On the other side, the new Isogeometric analysis approach (IgA), establishes a strict relation between the geometry of the problem domain and the approximate solution representation, giving surprising computational advantages. Great part of IgA literature has been focused on FEMs and only recently the IgA approach has been introduced in the framework of BEMs. In this talk, we apply the Isogeometric concept to a particular BEM, the so called Symmetric Galerkin BEM, using different types of spline functions to represent both the boundary of the problem domain and the approximate solution. The computational advantages over BEM based on Lagrangian basis will be underlined.

Bridging Frame Erasures [M-16B]

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In this talk, I will discuss a new method for reconstruction from frame coefficient erasures that is more efficient than older methods. This method, called Nilpotent bridging requires only an $L \times L$ matrix inversion, where older methods require an $n \times n$ matrix inversion to recover an n-dimensional signal from L frame coefficient erasures. To reconstruct from erasures indexed by Λ, the method uses information from a subset Ω of the known coefficients, known as the bridge set. I will discuss the existence of a bridge set for which $|\Omega| \leq |\Lambda|$. I will also show that for an open dense subset of the set of frames, any bridge set Ω, for which $|\Omega| = |\Lambda| \leq \min\{N - n, n\}$ will work for the reconstruction, and thus a bridge set search is an unnecessary step in the reconstruction algorithm. This is joint work with David Larson.

Exact Recovery from Wigner-D Moments on the Rotation Group [M-11B]

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In this talk we show the possibility of recovering a sum of Dirac measures on the rotation group from its low degree moments with respect to Wigner-D functions. Exact recovery is possible, if the support of the measure obeys a certain separation condition. The main ingredients for the proof are localization estimates for kernels on the rotation group.

The GLT Class as a Generalized Fourier Analysis and Applications [M-15B]

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Recently, the class of Generalized Locally Toeplitz (GLT) sequences has been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of the class, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum in terms of a function (the symbol) that can be easily identified. This generalizes the notion of a symbol for differential operators (discrete and continuous) or for Toeplitz sequences for which it is identified through the Fourier coefficients and is related to the classical Fourier analysis. The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT class virtually includes any approximation of partial differential equations (PDEs) by local methods (finite difference, finite element, isogeometric analysis, etc.) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE setting in various directions, including preconditioning, multigrid, spectral detection of branches, stability issues. We will discuss specifically the spectral potential of the theory with special attention to the isogeometric analysis setting.

Stable Radial Basis Function Methods for Meshfree Transport on the Sphere and Other Surfaces [M-1A]

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The transport of proteins and other molecules on cell membranes and other surfaces is of increasing interest in the biological and material sciences. In this talk, we present several novel numerical methods for simulating transport on surfaces. These methods are built on Radial Basis Function (RBF) interpolation. As is typical of RBF methods, our methods work purely with Cartesian coordinates, avoiding any coordinate singularities associated with intrinsic coordinate systems on manifolds. However, while current RBF methods for transport require careful tuning for stability, our new methods are self-stabilizing. We present results showing high orders of spatial convergence for transport on the sphere, and demonstrate the ability of some of our methods to handle transport on other surfaces.

On a Problem of Pinkus and Wajnryb Regarding Density of Multivariate Polynomials [M-1B]

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In this talk we will present an to answer to two questions posed by Allan Pinkus and Bronislav Wajnryb regarding density of certain classes of multivariate polynomials.

Stability of the Elastic Net Estimator [M-20B]

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The elastic net is a regularized least squares regression method that has been widely used in learning and variable selection. The elastic net regularization linearly combines an l_1 penalty term (like the lasso) and an l_2 penalty term (like ridge regression). The l_1 penalty term enforces sparsity of the elastic net estimator, whereas the l_2 penalty term ensures democracy among groups of correlated variables. Compressed sensing is currently an extensively studied technique for efficiently reconstructing a sparse vector from much fewer samples/observations. In this talk we discuss the elastic net in the setting of sparse vector recovering. For recovering sparse vectors from few observations by employing the elastic net regression, we prove that the elastic net estimator is stable provided that the underlying measurement/design matrix satisfies the commonly required restricted isometry property or the sparse approximation property/robust null space property. It is well known that many independent random measurement matrices satisfy the restricted isometry property while random measurement matrices generated by highly correlated Gaussian random variables satisfy the sparse approximation property/robust null space property. As a byproduct, we establish a uniform bound for the grouping effect of the elastic net.

Sparse Approximation using $\ell 1 - \ell 2$ Minimization and its Application to Stochastic Collocation [M-11A]

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We propose a sparse approximation method using $\ell 1 - \ell 2$ minimization. We present several theoretical estimates regarding its recoverability for both sparse and non-sparse signals. Then we apply the method to sparse orthogonal polynomial approximations for stochastic collocation. Various numerical examples are presented to verify the theoretical findings. We observe that the $\ell 1 - \ell 2$ minimization seems to produce results that are consistently better than the $\ell 1$ minimization.

On Construction of Multivariate Tight Wavelet Frames [M-8B]

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Construction of compactly supported tight wavelet frames in the multivariate case is much more complicated than in the univariate case because an analog of the Riesz Lemma does not exist for the trigonometric polynomials of several variables. We give a new method for the construction of compactly supported tight wavelet frames in $L_2(\mathbb{R}^d)$ with any preassigned approximation order n for arbitrary matrix dilation M. The number of wavelet functions generating a frame constructed in this way is less or equal to $(d+1)|\det M| - d$. Earlier Bin Han proposed a method, where the number of generating wavelet functions is less than $\left(\frac{3}{2}\right)^d |\det M|$. Another advantage of our method is in its simplicity. The method is algorithmic, and the algorithm is simple to use, it consists mainly of explicit formulas. Computations are needed only to find several trigonometric polynomials of one variable from their squared magnitudes. The number of generating wavelet functions can be reduced for a large enough class of matrices M. Namely, if all entries of some column of M are divisible by $|\det M|$, then the algorithm can be simplified so that the number of wavelet functions does not exceed $|\det M|$. Moreover, the existence of compactly supported tight wavelet frames with $|\det M| - 1$ wavelet functions and an arbitrary approximation order is proved for such matrices.

Bivariate Spline Solution of Time Dependent Nonlinear PDE for Population Density over Irregular Domains [M-3B]

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We present a method of solving a time dependent partial differential equation, which arises from classic models in ecology concerned with a species' population density over two dimensional domains. The species experiences population growth and diffuses over time due to overcrowding. Population growth is modeled using logistic growth with Allee effect. We introduce the concept of discrete weak solution and establish theory for the existence, uniqueness and stability of the solution. We use bivariate splines of arbitrary degree and smoothness across elements to approximate the discrete weak solution. More recent efforts focus on modeling the interaction of multiple species, which either compete for a common resource or one predates on the other. We present some simulations of population development over some irregular domains.

Function Approximation from Non-uniform Fourier Data [M-3A]

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Non-uniform Fourier data are routinely collected in applications such as magnetic resonance imaging (MRI), synthetic aperture radar (SAR), and synthetic imaging in radio astronomy. We will discuss in this talk the approximation of an unknown function from its finite non-uniform Fourier samples. Specifically, we provide a mathematical formulation of this problem through the use of Fourier frames and the development of admissible frames. As a result, a stable and efficient algorithm based on that will also be presented.

Bernstein-Bézier Techniques for Spline Vector Fields [M-1B]

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We consider the application of standard differentiation operators to spline spaces and spline vector fields defined on simplicial partitions in $\mathbb{R}^n$. In particular, we develop Bernstein-Bézier techniques for working with divergence of spline vector fields. Using the new Bernstein-Bézier techniques, we obtain the dimension and a minimal determining set for divergence-free splines on Alfeld split of a simplex in $\mathbb{R}^n$. If time permits, we will apply the techniques to prove a version of a generalized Stokes' Theorem for continuous splines.

Semialgebraic Splines [M-1B]

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Semialgebraic splines are functions that are piecewise polynomial with respect to a cell decomposition of their domain into sets defined by polynomial inequalities. We study bivariate semialgebraic splines, computing the the dimension of the space of splines with large degree in two extreme cases when the cell decomposition has a single internal vertex. First, when the forms defining the edges span a two-dimensional space of forms of degree n—then the curves they define meet in n^2 points in the complex projective plane. In the other extreme, the curves have distinct slopes at the vertex and do not simultaneously vanish at any other point. This is joint work with Michael DiPasquale and Lanyin Sun.

Effortless Quasi-interpolation in Hierarchical Spline Spaces [M-13B]

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Hierarchical spline spaces provide a flexible framework for local refinement coupled with a remarkable intrinsic simplicity. They are defined in terms of a hierarchy of locally refined meshes, reflecting different levels of refinement. The so-called truncated hierarchical basis is an interesting basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical basis: its elements form a convex partition of unity, they are locally supported and strongly stable. In this talk we discuss a general approach to construct quasi-interpolants in hierarchical spline spaces expressed in terms of the truncated hierarchical basis. The main ingredient is the property of preservation of coefficients of the truncated hierarchical basis representation. Thanks to this property, the construction of the hierarchical quasi-interpolant is basically effortless. It is sufficient to consider a quasi-interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one-level quasi-interpolant. Then, the coefficients of the proposed hierarchical quasi-interpolant are nothing else than a proper subset of the coefficients of the one-level quasi-interpolants. No additional manipulations are required. Important properties – like polynomial reproduction – of the one-level quasi-interpolants are preserved in the hierarchical construction. We also discuss the local approximation order of the hierarchical quasi-interpolants in different norms, and we illustrate the effectiveness of the approach with some numerical examples.

Potential Theoretic Approach to Design of Approximation Formulas in Symmetric Weighted Hardy Spaces [C-10B]

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We propose a method for designing accurate interpolation formulas on the real axis for the purpose of function approximation and numerical integration in weighted Hardy spaces. In particular, we consider the Hardy space of functions that are analytic in a strip region around the real axis, being characterized by a weight function w that determines the decay rate of its elements in the neighborhood of infinity. Such a space is considered as a set of functions that are transformed by variable transformations that realize a certain decay rate at infinity. So far, interpolation by sinc functions is a representative way for approximating the functions in the space. However, it is not guaranteed that the sinc formulas are optimal. Then, we adopt a potential theoretic approach to obtain almost optimal formulas in weighted Hardy spaces in the case of general weight functions w. We formulate the problem of designing an optimal formula in each space as an optimization problem written in terms of a Green potential with an external field. By solving the optimization problem numerically, we obtain an almost optimal formula that outperforms the sinc formulas. Furthermore, we applied the approach to obtain accurate formulas for numerical integration. This is a joint work with Dr. Tomoaki Okayama (Hiroshima City University) and Prof. Masaaki Sugihara (Aoyama Gakuin University).

Dynamical Sampling in Hilbert Space [M-16B]

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Let $f \in \ell^2(I)$ be a signal at time $t = 0$ of an evolution process controlled by a bounded linear operator A that produces the signals $Af, A^2 f, \cdots$ at times $t = 1, 2, \cdots$. Let $Y = \{f(i), Af(i), \cdots, A^{l_i} f(i) :$

$i \in \Omega \subset I\}$ be the spatio-temporal samples taken at various time levels. The problem under consideration is to find necessary and sufficient conditions on A, Ω, l_i in order to recover any $f \in \ell^2(I)$ from the measurements Y. This is the so called Dynamical Sampling Problem in which we seek to recover a signal f by combining coarse samples of f and its futures states $A^l f$. In this talk, we will study this problem and show some recent results.

Stable Extrapolation of Analytic Functions [M-6B]

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Mathematical folklore states that polynomial extrapolation of a function is hopeless, especially when only equally-spaced function samples are known.In this talk we explain how a more precise statement carries an interesting nuance. Provided a standard oversampling condition is satisfied, we give a recipe for constructing an asymptotically best extrapolant as a piecewise polynomial. Along the way we derive explicit bounds for the quality of least squares approximations from equally-spaced samples, robustness to perturbed samples, and a faster direct algorithm for constructing least squares approximants.

Some Reductions and Equivalences of the Lower Bound Formula [M-1B]

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The lower bound formula (also known as Strang's conjecture) characterizes the dimension of bivariate spline spaces $S_3^1(\Delta)$ based on the number of boundary vertices, interior vertices, and singular vertices. The conjecture seems true according to all evidence, but the proof remains stubbornly elusive. In this talk we sketch some equivalent formulations of Strang's conjecture, including reductions to simpler Δ, using a dual perspective on splines.

Data-driven Atomic Decomposition via Frequency Extraction of Intrinsic Mode Functions [M-11B]

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Decomposition of signals into finitely many primary building blocks, called atoms, is a fundamental problem in signal processing, with many potential high-impact applications, particularly when real-world signals or time series are considered. The goal of this talk is to describe the construction of atoms for signal decomposition directly from the input signal data, using local methods. The highlights of our discussion include a natural formulation of the data-driven atoms, a modified sifting process (based on the popular empirical mode decomposition (EMD)) for real-time implementation, further decomposition of the intrinsic mode functions, motivation of the signal separation operator, and the problem of super-resolution, leading to our new computational scheme for data-driven atomic decomposition, which we've coined "superEMD".

Harmonic Empirical Mode Decomposition Method [C-5A]

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N. Huang et al. introduced the Empirical Mode Decomposition Method (EMD) and the Huang

Hilbert Transform in 1998 to analyze nonstationary and nonlinear signals. The method has been proven to be very effective in a great variety of applications. The development of the theory of EMD, including convergence and alternative characterization of the decomposition modes, has lead to numerous EMD modifications and new and effective algorithms for decomposition of functions, among which the Synchrosqueezed wavelet transforms: An empirical mode decomposition-like tool, introduced in 2011 by Daubechies et al. The method is considered a departure from the classical Fourier series but in the talk we show how the main components of the original EMD, the scale and the envelopes through the extrema, can be related to bi-orthogonal Fourier series expansions with varying phases. Building on that analogy we introduce the Harmonic EMD and provide results about its convergence and characterize the outcome modes. The performance of HEMD is compared to the performance of the classical EMD in several numerical examples.

Geometrically Continuous Splines on Surfaces of Arbitrary Topology [M-13B]

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We study the space of geometrically continuous splines, or piecewise polynomial functions, on topological surfaces. These surfaces consist of a collection of rectangular and triangular patches together with gluing data that identifies pairs of polygonal edges. A spline is said to be G^1-geometrically continuous on a topological surface if they are C^1-continuous functions across the edges after the composition by a transition map. In the talk we will describe the required compatibility conditions on the transition maps so that the C^1-smoothness is achieved, and give a formula for a lower bound on the dimension of the G^1 spline space. In particular, we will show that this lower bound gives the exact dimension of the space for a sufficiently large degree of the polynomials pieces. We will also present some examples to illustrate the construction of basis functions for splines of small degree on particular topological surfaces.

Variable Selection for Semi-parametric Geospatial Models [M-3B]

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In this paper, we focus on the variable selection techniques for a class of semi-parametric geospatial models which allow one to study the effect of the covariates in the presence of the spatial information. The smoothing problem in the nonparametric part is tackled by means of bivariate splines over triangulation, which is able to deal efficiently with data distributed over irregularly shaped regions. In addition, we develop a unified procedure for variable selection to identify significant linear covariates under a double penalization framework, and we show that the penalized estimators enjoy the oracle property. The proposed method can simultaneously identify non-zero covariates in the linear part and solve the problem of "leakage" across complex domains of the nonparametric part. To estimate the standard deviations of the proposed estimators for the coefficients, a sandwich formula is developed as well. In the end, several simulation examples and a real data analysis are studied to illustrate the proposed methods.

Bivariate Penalized Splines for Geo-Spatial Regression [M-3B]

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We study the estimation of partially linear models for spatial data distributed over complex domains. We use bivariate splines over triangulations to represent the nonparametric component on an irregular 2D domain. This method does not require constructing finite elements or locally supported basis functions, allowing an easier implementation of piecewise polynomial representations of various degrees. A penalized least squares method is proposed to estimate the model via QR decomposition. The estimators of the parameters are proved to be asymptotically normal under some regularity conditions. The estimator of the bivariate function is consistent, and its rate of convergence is also established. The proposed method enables us to construct confidence intervals and permits inference for the parameters. The performance of the estimators is evaluated by several simulation examples and a real data analysis.

Fast Framelet Transforms on Manifolds [M-3A]

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Tight framelets on a compact Riemannian manifold $\mathcal{M}$ provide a tool of multiresolution analysis for data on graphs, geosciences and astrophysics. This work, joint with Xiaosheng Zhuang, describes an efficient digital implementation of the framelet transform on $\mathcal{M}$ — the fast framelet transform (FMT) algorithm. The FMT algorithm, using the filter bank and the quadrature rule of the framelets, has the computational complexity $\mathcal{O}(N)$, where N is the number of input data. We give examples of FMT for spheres and tori.

Inverse Estimates for Compact Domains in R^d using Localized Kernel Bases [M-1A]

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This talk will discuss inverse estimates for finite dimensional spaces arising in radial basis function approximation and meshless methods. The inverse estimates we consider control Sobolev norms of linear combinations of a localized basis by the L_p norm over a bounded domain. These estimates are valid for Matern and polyharmonic families of radial basis functions. The localized basis is generated by forming certain local Lagrange functions.

Polynomial Approximation via Compressed Sensing of High Dimension Complex-valued Functions [M-13A]

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In this talk, we present a compressed sensing approach to polynomial approximation of complex-valued functions in high dimensions. Of particular interest is the parameterized PDE setting, where the target function is smooth, characterized by a rapidly decaying orthonormal expansion, whose most important terms are captured by a lower (or downward closed) set. By exploiting this fact, we develop a novel weighted ℓ_1 minimization procedure with a precise choice of weights, and a modification of the iterative hard thresholding method, for imposing the downward closed preference. We will also present theoretical results that reveal our new computational approaches possess a provably reduced sample complexity compared to existing compressed sensing, least squares, and interpolation techniques. In addition, the recovery of the corresponding best approximation using our methods is established through an improved bound for the restricted isometry property. Numerical examples are provided to support the theoretical results and demonstrate the computational efficiency of the new weighted ℓ_1 minimization method.

Interpolation by Transformed Snapshots [M-6A]

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Parametric and stochastic hyperbolic PDEs pose challenging approximation problems: Because of the possibly random parameters the solutions are high dimensional and in addition the prevalence of shock discontinuities diminishes the convergence rates of virtually all established methods. A simple method underlying many numerical schemes is a simple interpolation (in parameter) of snapshots, i.e. solutions of the parametric PDE for fixed parameters. In the hyperbolic context, this method suffers from discontinuities in parameter induced by the shocks of the solutions. To this end, for each new target parameter, I propose to use a specifically tailored set of transformed snapshots in the interpolation procedure. These transformations serve the purpose to increase the regularity in parameter direction by aligning the shocks so that the interpolation procedure yields significantly improved convergence rates. They are automatically computed in an offline phase and can be efficiently applied during function evaluation. The method is tested for some 2d problems.

Optimal Complexity Spectral Methods for Partial Differential Equations on the Sphere and Disk [M-8A]

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Spherical harmonics and Bessel functions are frequently employed bases in spectral methods for solving elliptic partial differential equations (PDEs) defined on the unit sphere and disk, respectively. We discuss faster and more convenient spectral methods for such PDEs based on Fourier and ultraspherical bases. Optimal complexity and numerically stable solvers for the Poisson and Helmholtz equations are described that achieve essentially uniform resolution for the sphere and disk. We incorporate these with a semi-implicit time-stepping scheme to efficiently solve various time-dependent PDEs, including reaction-diffusion equations on the sphere and the Navier-Stokes equations on the disk. We will also discuss the new Spherefun and Diskfun software that contains seamless access to these new solvers.

Isogeometric Analysis with Rational Triangular Bézier Splines [M-13B]

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In this talk, we present an approach of isogeometric analysis with rational triangular Bézier splines (rTBS) where optimal convergence rates are achieved. In this approach, both the geometry and the physical field are represented by bivariate (or trivariate for 3D geometries) splines in Bernstein Bézier form over the triangulation of the geometry. From a given physical domain bounded by NURBS curves or surfaces, a parametric domain and its triangulation are constructed. By imposing continuity constraints on the Bézier ordinates, we obtain global C^r smoothness over the triangular mesh. Moreover, using the macro-element technique, we construct a set of global C^r smooth basis functions that are linearly independent and locally supported. Convergence analysis shows that isogeometric analysis with such C^r rTBS basis can deliver the optimal rates of convergence provided that the C^r geometric map remains unchanged during the refinement process. This condition can be satisfied by constructing a pre-refinement geometric map that is sufficiently smooth. Numerical experiments verify that optimal rates of convergence are achieved for both 2D and 3D geometries.

Analysis Recovery with Coherent Dictionaries and Correlated Measurements [M-3A]

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In practical examples, there are numerous signals of interest to be sparse with a redundant dictionary. It helps to add a lot of flexibility and significantly extends the range of applicability. In this talk, we consider analysis compressed sensing with coherent frames and correlated measurements. We mention the notation of the Restricted Eigenvalue condition adapted to general Frame D, which is a natural extension to the standard Restricted Eigenvalue condition. We establish the D-RE condition for several classes of correlated measurement matrices, when the covariance matrix of row measurements satisfies the D-RE condition. Furthermore, by the D-RE condition, we get the error bounds in the analysis LASSO (ALASSO) and the analysis Dantzig Selector (ADS) under a sparsity scenario.

In order to recover non-sparse signals, we consider the robust D-Nullspace property of correlated Gaussian matrices. Similarly, we get the error estimations in the ALASSO and the ADS in non-sparse case. The approximation equivalence between the ALASSO and the ADS is also established with robust D-Nullspace property.

Real Zeros of Random Orthogonal Polynomials [M-16A]

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We study the expected number of real zeros for random linear combinations of orthogonal polynomials. It is well known that Kac polynomials, spanned by monomials with i.i.d. Gaussian coefficients, have only $(2/\pi + o(1)) \log n$ expected real zeros in terms of the degree n. If the basis is given by the orthonormal polynomials associated with a compactly supported Borel measure on the real line or associated with a Freud weight defined on the whole real line, then random linear combinations have $n/\sqrt{3} + o(n)$ expected real zeros. We also prove that the same asymptotic relation holds for all random orthogonal polynomials on the real line associated with a large class of exponential weights. This reveals the universality of the expected number of real zeros for random orthogonal polynomials. This talk is based on joint work with Doron Lubinsky and Igor Pritsker.

Approximation and Orthogonality in the Sobolev Space on a Triangle [M-6B]

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The best approximation polynomials in L^2 space are the partial sums of the Fourier orthogonal expansions in the L^2 space. This holds, in some sense, also for the Sobolev space, for which the orthogonality is defined with respect to an inner product that contains derivatives. We explain the methodology and the result on an interval and on a triangle in this talk.

Numerical Differentiation By Kernel-based Probability [M-8A]

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In this talk, we show a proposed approximate method of numerical differentiation by a new concept of kernel-based probability. We combine techniques in meshfree method and stochastic regression to recover high-dimensional derivative for scattered noise data. A positive definite kernel is a proven tool to construct a kernel-based probability measure on a Sobolev space such that normal random variables endowed with kernel-based probability distributions are well-defined by smooth functions with derivatives. This allows that Bayesian estimation method is used to formulate kernel-based approximant for numerical differentiation by the normal random variables. Moreover, error analysis and convergent rate of the proposed kernel-based approximant are given by conditional mean square errors of the normal random variables such as posteriori error bounds. A special example of Gaussian kernels in three dimensions is included to numerically verify effectiveness and stability of the novel kernel-based formulations.

Zeros of Random Linear Combinations of Entire Functions with Complex Gaussian Coefficients [M-18A]

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Recently Vanderbei, extending his previous work with Shepp, gave an explicit formula for the expectation of the number, $N_n(\Omega)$, of zeros of a random sum

$$P_n(z) = \sum_{j=0}^{n} \eta_j f_j(z),$$

in any measurable set $\Omega \subset \mathbb{C}$. Here the functions f_j are given entire functions that are real-valued on the real line, and $\eta_0, \ldots, \eta_n$ are real-valued independent standard normal random variables. In this talk, following a similar method we derive an explicit formula for a class of random sums with complex Gaussian coefficients. We will also show that when the entire functions are taken to be polynomials orthogonal on the real line, using the Christoffel-Darboux Formula the intensity function takes a very simple looking shape. We will conclude the talk by considering the limiting intensity function.

Compactly Supported Parseval Framelet with Symmetry [M-8B]

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Let $d \geq 1$. For any $A \in \mathbb{Z}^{d \times d}$ such that $|\det A| = 2$, we construct two families of Parseval wavelet frames with two generators. These generators have compact support, any desired number of vanishing moments, and any given degree of regularity. The first family is real valued while the second family is complex valued. To construct these families we use Daubechies low pass filters to obtain refinable functions, and adapt methods employed by Chui and He and Petukhov for dyadic dilations to this more general case. We also construct several families of Parseval wavelet frames with three generators having various symmetry properties. Our constructions are based on the same refinable functions and on techniques developed by Han and Mo and by Dong and Shen for the univariate case with dyadic dilations.

A Fast and Efficient Helmholtz Solver via Adaptive Learning of the Basis [M-6B]

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We present a ray-based finite element method (Ray-FEM) for the high-frequency Helmholtz equation in smooth acoustic media. The method relies on a local geometrical optics Anzat coupled with an adaptive learning of the basis. The learning phase consists on solving the Helmholtz equation at low-frequency to extract the local ray direction of the wavefield. The ray directions are then incorporated into the basis, which are used to solve the high-frequency problem. The process can be continued to further improve the approximation for both the ray directions and the high frequency wavefield iteratively. The method only requires the minimum degrees of freedom, i.e., a fixed number of grid points per wavelength, to achieve both stability and expected accuracy for the high-frequency Helmholtz solution without the usual pollution effect. Under some assumptions, the resulting matrices can be solved using fast methods in an empirical complexity $\mathcal{O}(\omega^d)$, where ω is the frequency. Numerical tests in 2D are presented to corroborate the claims.

A Multilevel Reduced-basis Method for Parameterized Partial Differential Equations [M-11A]

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An important approximation scheme for alleviating the overall computational complexity of solving parameterized PDEs is known as multilevel methods, which have been successfully used in the Monte Carlo and collocation setting. In this effort, we propose to improve the multilevel methods with the use of reduced-basis (RB) techniques for constructing the spatial-temporal model hierarchy of PDEs. Instead of approximating the solution manifold of the PDE, the key ingredient is to build approximate manifolds of first-order differences of PDE solutions on consecutive levels. To this end, we utilize a hierarchical finite element (FE) framework to formulate an easy-to-solve variational FE system for the first-order differences. Moreover, by deriving a posteriori error estimates for the RB solutions, we also intend to develop a greedy-type adaptive strategy in order to construct a good set of snapshots. The main advantage of our approach lies in the fact that the manifold of the first-order differences becomes progressively linear as the physical level increases. Thus, much fewer expensive snapshots are required to achieve a prescribed accuracy, resulting in significant reduction of the offline computational cost of greedy algorithms. Furthermore, our approach combines the advantages of both multilevel Monte Carlo and multilevel collocation methods, in the sense that it can generate snapshots anywhere in the parameter domain but also features fast convergence.

Recent Advances in Volumetric T-spline Parameterization: Basis Functions [M-15B]

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In this talk, I will show our latest research on volumetric T-spline parameterization - basis function development, which contributes directly to the integration of design and analysis, the root idea of isogeometric analysis. Weighted and truncated T-spline basis functions are derived to satisfy the requirements of analysis-suitable T-splines. Partition of unity is enabled via introducing a new weight calculation scheme or truncation mechanism. Knot interval duplication is adopted to infer knot vectors for the vicinity around extraordinary nodes, based on which bicubic weighted T-spline basis functions are defined. Volumetric T-splines are constructed using the parametric mapping and sweeping methods, which have been incorporated with commercial software packages such as Rhino and Abaqus.

Adaptive RBF Interpolation Achieved by Error Indicator [M-1A]

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In some approximation problems sampling from the target function can be both expensive and time-consuming. It would be convenient to have a method for indicating where approximation quality is poor, so that generation of new data provides the user with greater accuracy where needed. We propose a new adaptive algorithm for radial basis function (RBF) interpolation which aims to assess the local approximation quality, and add or remove points as required to improve the error in the specified region. Numerical results for test functions are given for dimensions 1 and 2, to show that our method performs well. We also give a 3 dimensional example from the finance world, since we would like to advertise RBF techniques as useful tools for approximation in the high dimensional settings one often meets in finance. Also we applied our technique to a practical industrial 3D case with the engineers, which the results shows that our technique is highly helpful in reducing the resources that the engineers needs. The condition numbers of interpolation matrices have been keep low in order to delivery reliable approximation according to the engineers need.

Optimal RIP Bounds for Sparse Signals Recovery via ℓ_p Minimization [M-20B]

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we present a unified analysis of RIP bounds for sparse signals recovery by using ℓ_p minimization with $0 < p \leq 1$ and provide optimal RIP bounds which can guarantee sparse signals recovery via ℓ_p minimization for each $p \in (0, 1]$ in both noiseless and noisy settings. It is shown that if the measurement matrix Φ satisfies the RIP condition $\delta_{2k} < \delta(p)$, where $\delta(p)$ will be specified in the context, then all k-sparse signals x can be recovered stably via the constrained ℓ_p minimization based on $b = \Phi x + z$, where z is one type of noise. Furthermore, we show that for any $\epsilon > 0$, $\delta_{2k} < \delta(p) + \epsilon$ is not sufficient to guarantee the exact recovery of all k-sparse signals. We also apply the results to the cases of low rank matrix recovery and the reconstruction of sparse vectors in terms of redundant dictionary.

Multilevel Sparse Grid Kernels Based Collocation in Linear PDE Solving [C-5B]

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we apply the multilevel sparse grid kernels (MuSIK) algorithm into collocation method (refer to as MuSIK-C) to solve elliptic and parabolic problems. Here we follow Myers et al.'s suggestion, considering time as one spatial dimension. Using previous algorithms to solve time depending partial differential equation (PDE) with radial basis functions (RBFs), specific discretization on time direction is essential. Due to the specific discretization on time direction, it is inconvenient to have approximations at points outside the discretization grid. Having an efficient time and spatial grid is crucial to the approximation accuracy in major previous algorithm, however it is not clear which direction has more effect on the approximation. This suggestion could avoid the above disadvantages and reduce the complexity. For example, solving a $\mathbb{T} \times \mathbb{R}^d$ PDE with MuSIK-C as usual, we may need $\mathcal{O}(N^2 \log^{d-1}(N))$ operations, where N is nodes number in each direction. However, if consider time as one dimension, the complexity is $\mathcal{O}(N \log^d(N))$. In numerical experiments with multiquadric and Gaussian RBFs, MuSIK-C gives rapid convergency even under 4 dimensions ($\mathbb{T} \times \mathbb{R}^3$).

Gradient Enhanced Stochastic Collocation Methods for UQ [M-13A]

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We consider stochastic collocation methods for high dimensional parametric functions. In particular, gradient information will be included together with function values, and we consider approaches such us L^1 minimization and lease squares.

Digital Affine Shear Transforms Fast Realization and Applications [M-3A]

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In this talk, we mainly discuss the digitization and applications of the recent developed directional multiscale representation systems: smooth affine shear tight frames. An affine wavelet tight frame is generated by isotropic dilations and translations of directional wavelet generators, while an affine shear tight frame is generated by anisotropic dilations, shears, and translations of shearlet generators. These two tight frames are actually connected in the sense that the affine shear tight frame can be obtained from an affine wavelet tight frame through subsampling. Consequently, an affine shear tight frame indeed has an underlying filter bank from the MRA structure of its associated affine wavelet tight frame. We discuss the digitization of digital affine shear filter banks associated with the affine shear tight frames in arbitrary dimension $d \geq 2$. Moreover, we discuss the redundancy rate and computational complexity of our digital affine shear transforms, which conclude that the redundancy rate of our digital affine shear transforms does not increase with respect to the number of directions and the computational complexity is proportional to the redundancy rate and the FFT times for a fixed size of input data. Numerical experiments and comparisons in image/video processing show the advantages of digital affine shear transforms over many other state-of-the-art frame-based directional transforms.

Truncated B-splines for Partially Nested Refinement [M-13B]

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Motivated by the necessity to perform local refinement in geometric design and numerical simulations - especially in the context of Isogeometric Analysis - various approaches to extend the construction of tensor-product splines have been introduced: T-splines, locally-refined splines and (truncated) hierarchical (TH)B-splines. Our research is based on THB-splines (and their generalizations) (Kraft, 1997; Giannelli et al., 2012; Zore and Jüttler, 2014), as they possess advantageous properties, such as linear independence, the partition of unity property, preservation of coefficients and strong stability and sparsity properties, which make them highly useful in various applications (Giannelli et al., 2016). The construction of THB-splines uses two essential ingredients: a nested sequence of subdomains (which describe the regions identified for further refinement), together with a *nested* sequence of spline spaces. Due to the latter requirement, the existing hierarchical framework does not allow for unrelated refinements in different parts of the model. In order to overcome this limitation, we discuss a possible generalization of the hierarchical model, where we consider a sequence of tensor-product spline spaces, which are only partially nested. *Truncated B-splines for partially nested refinement* are then defined by their local representations on patches, which partition the parameter domain. Under certain conditions, they are shown to be linearly independent, form a non-negative partition of unity, possess the preservation of coefficients property, and inherit the smoothness of the underlying splines spaces. Moreover, the construction results in the standard THB-splines, provided that the spaces form a nested sequence.