1. [1 each] Determine whether or not the following statements are true or false. If they are **always** true circle **T**. If they are not **always** true, circle **F**.

   (a) **T** **F** $\int_{-5}^{5} \frac{x^2 \sin x}{1+\cos^2 x} dx = 0.$ 5.4.41

   (b) **T** **F** For any constant $b > 0$, the area of the region between $y = be^x$ and $y = \ln x$ on [1,2] is $\int_1^2 (be^x - \ln x)dx$. Use $|be^x - \ln x|$ instead. 6.2.39b

   (c) **T** **F** The displacement of a particle may be negative, but the distance traveled by a particle cannot be negative. Def of "displacement" & "distance"

   (d) **T** **F** The average value of a linear function on an interval is the value of the function at the midpoint of the interval. 5.4.37c

   (e) **T** **F** $\int_a^b \sqrt{1+f'(x)^2}dx = \int_a^b (1+f'(x))dx.$ $\sqrt{p^2+q^2} \neq p+q$ 6.5.25a

2. [10] Let $f(x) = \frac{e^{2x}}{1+e^{2x}}$ on the interval $[0, \frac{1}{2}\ln 3]$. (5.4.31)

   (a) Find the average value of $f(x)$ over the interval.

$$\frac{1}{\frac{1}{2}\ln 3 - 0}\int_0^{\frac{1}{2}\ln 3} \frac{e^{2x}}{1+e^{2x}}dx = \frac{1}{\ln 3}\int_2^4 \frac{du}{u} = \frac{1}{\ln 3}\ln\frac{4}{2}$$

(subst. $u = 1+e^{2x}$)
($du = 2e^{2x}dx$)

$$= \boxed{\frac{\ln 2}{\ln 3}} = \boxed{\log_3 2}$$

(check: $\frac{d}{dx}\ln(1+e^{2x}) = \frac{2e^{2x}}{1+e^{2x}}$)

   (b) Find the values c in the interval where $f(c)$ equals the average value.

$$\frac{e^{2c}}{1+e^{2c}} = \log_3 2$$

$$e^{2c} = (1+e^{2c})\log_3 2$$

$$e^{2c}\log_2 3 = 1 + e^{2c}$$

$$e^{2c}(\log_2 3 - 1) = 1$$

$$e^{2c} = \frac{1}{\log_2 3 - 1}$$

$$2c = \ln\left(\frac{1}{\log_2 3 - 1}\right)$$

$$\boxed{c = \frac{1}{2}\ln\left(\frac{1}{\log_2 3 - 1}\right)}$$

$$\boxed{c = -\frac{1}{2}\ln\left(\log_2 \frac{3}{2}\right)}$$