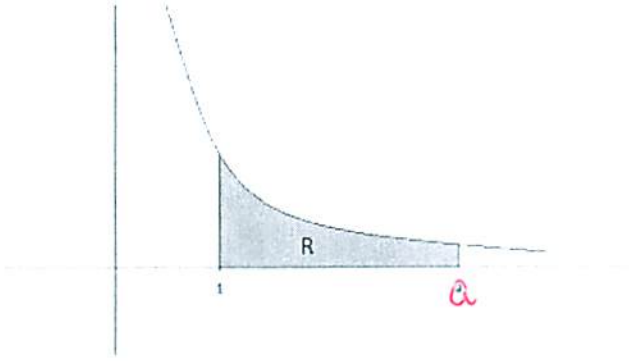
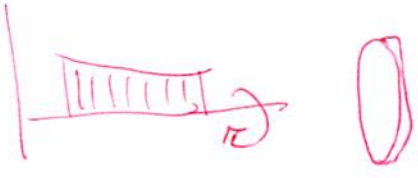
4. [21] Let R be the region bounded by the curve $y = 1/x$, the x-axis, $x = 1$, and $x = a > 1$.



(a) Find the area of R.

$$\int_1^a \frac{1}{x}\,dx = \ln a - \ln 1 = \boxed{\ln a} \qquad (5.5.65)$$

(b) Find the volume of the solid of revolution obtained by revolving R around the x-axis.

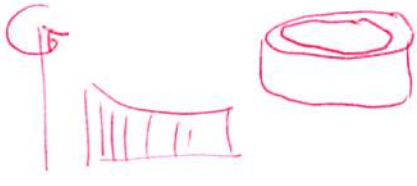


$$\int_1^a \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^a x^{-2}\,dx$$

$$= -\pi\left[x^{-1}\right]_1^a = \boxed{\pi\left(1 - \frac{1}{a}\right)}$$

(6.3.17 or shells)

(c) Find the volume of the solid of revolution obtained by revolving R around the y-axis.



$$\int_1^a 2\pi x \cdot \frac{1}{x}\,dx = \int_1^a 2\pi\,dx$$

$$= \boxed{2\pi(a-1)}$$

(6.3.31 or shells)

(d) Find the limits as $a \to \infty$ of the volumes found in parts (b) and (c).

For part b, limit is $\boxed{\pi}$

For part c, limit is $\boxed{\infty}$