3. [29] Evaluate the following integrals.

(a) $\int \cos x \csc^{100} x \, dx$ $= \int u^{-100} du = \dfrac{u^{-99}}{-99} + C$

$u = \sin x$

$du = \cos x \, dx$

$= \boxed{-\dfrac{1}{99} \csc^{99} x + C}$

(5.5. 15, 17)

(b) $\int_{1/10}^{\sqrt{2}/10} \frac{1}{\sqrt{1-25x^2}} \, dx$ $= \dfrac{1}{5} \int_{1/2}^{\sqrt{2}/2} \dfrac{1}{\sqrt{1-u^2}} du = \dfrac{1}{5} \left[\sin^{-1} u \right]_{\frac{1}{2}}^{\frac{1}{2}\sqrt{2}}$

$u = 5x$

$du = 5dx$

$\frac{1}{5} du = dx$

$= \dfrac{1}{5} \left[\dfrac{\pi}{4} - \dfrac{\pi}{6} \right] = \boxed{\dfrac{\pi}{60}}$

(5.5.25)

(c) $\int \frac{x+3}{\sqrt{x+2}} \, dx$ $= \int \dfrac{u+1}{\sqrt{u}} du = \int \left(u^{1/2} + u^{-1/2} \right) du$

$u = x+2$

$du = dx$

$= \dfrac{2}{3} u^{3/2} + 2u^{1/2} + C$

$= \boxed{\dfrac{2}{3} (x+2)^{3/2} + 2\sqrt{x+2} + C}$

(5.5. 29)

(d) $\int_1^2 \left[f(x) + 3(f(x))^2 + f''(x) \right] f'(x) \, dx$ where $f(1) = a$, $f(2) = b$, $f'(1) = c$, $f'(2) = d$

(5.5. 79, 81)

$= \int_1^2 \left[f + 3f^2 \right] f' dx + \int_1^2 f'' f' dx$

$\left(\begin{array}{l} v = \frac{1}{2} f^2 + f^3 \\ dv = (f + 3f^2) f' dx \end{array} \right)$ $\left(\begin{array}{l} f''(x) dx = du \\ f'(x) = u \end{array} \right)$

$= \int dv + \int u \, du = \left[v + \frac{1}{2} u^2 \right]$

$= \left[\frac{1}{2} f^2 + f^3 + \frac{1}{2} (f')^2 \right]_{x=1}^{x=2}$

$= \left[\frac{1}{2} f(2)^2 + f(2)^3 + \frac{1}{2} f'(2)^2 \right] - \left[\frac{1}{2} f(1)^2 + f(1)^3 + \frac{1}{2} f'(1)^2 \right]$

$= \boxed{\left[\frac{1}{2} b^2 + b^3 + \frac{1}{2} d^2 \right] - \left[\frac{1}{2} a^2 + a^3 + \frac{1}{2} c^2 \right]}$