

## Review 2.

Evaluate the following integrals.

1.

$$\int \frac{1}{2x^2 - 4x + 9} \, dx$$

**Solution:**  $\frac{1}{\sqrt{14}} \arctan \frac{\sqrt{2}(x-1)}{\sqrt{7}} + C$

2.

$$\int \frac{x - 5}{x^2 - 2x + 2} \, dx$$

**Solution:**  $\ln \sqrt{x^2 - 2x + 2} - 4 \arctan(x - 1) + C$

3.

$$\int \frac{x^3}{x^2 + x + \frac{1}{2}} \, dx$$

**Solution:**  $\frac{(x-1)^2}{2} + \frac{1}{4} \ln \left( x^2 + x + \frac{1}{2} \right) + C$

4.

$$\int \frac{1}{x(x^2 + 5)} \, dx$$

**Solution:**  $\frac{1}{5} \ln \sqrt{\frac{x^2}{x^2+5}} + C$

5.

$$\int \frac{1}{(x+2)^2(x+3)^3} \, dx$$

**Solution:**  $2 \ln \left| \frac{x+3}{x+2} \right| - \frac{1}{x+2} - \frac{1}{x+3} + C$

6.

$$\int \frac{1}{(x+1)^2(x^2+1)} \, dx$$

**Solution:**  $\frac{1}{2} \ln \left| \frac{x+1}{\sqrt{x^2+1}} \right| - \frac{1}{x+1} + C$

7.

$$\int \frac{1}{(x^2+2)^2} dx$$

**Solution:**  $\frac{1}{4} \left( \frac{x}{x^2+2} + \frac{1}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} \right) + C$

8.

$$\int \frac{1}{x^4 - 2x^2 + 1} dx$$

**Solution:**  $\frac{1}{4} \left( \frac{2x}{1-x^2} + \ln \left| \frac{x+1}{x-1} \right| \right) + C$

9.

$$\int \frac{3-4x}{(1-2\sqrt{x})^2} dx$$

**Solution:**  $\frac{x(3+2\sqrt{x})}{1-2\sqrt{x}} + C.$

10.

$$\int \frac{(\sqrt{x}+1)^2}{x^3} dx$$

**Solution:**  $-\frac{1}{x} - \frac{4}{3x\sqrt{x}} - \frac{1}{2x^2} + C$

11.

$$\int \frac{1}{\sqrt{x^2+x+1}} dx$$

**Solution:**  $\ln \left( x + \frac{1}{2} + \sqrt{x^2+x+1} \right) + C$

12.

$$\int \frac{1 - \sqrt[3]{2x}}{\sqrt{2x}} dx$$

**Solution:**  $\sqrt{2x} - \frac{3}{5}(2x)^{\frac{5}{6}} + C$

13.

$$\int \frac{1}{(\sqrt[3]{x^2} + \sqrt[3]{x})^2} dx$$

**Solution:**  $-\frac{3}{\sqrt[3]{x+1}} + C$

14.

$$\int \frac{2x+1}{\sqrt{(4x^2-2x+1)^3}} dx$$

**Solution:**  $\frac{2x-1}{\sqrt{4x^2-2x+1}} + C$

15.

$$\int \frac{x^2}{\sqrt{(x^2-1)^3}} dx$$

**Solution:**  $\ln|x + \sqrt{x^2-1}| - \frac{x}{\sqrt{x^2-1}} + C$

16.

$$\int \frac{x}{(1+x^2)\sqrt{1-x^4}} dx$$

*Hint.* If you arrive to the integral  $\int \frac{1}{1+\sin\theta} d\theta$ , multiply the integrand by  $\frac{1-\sin\theta}{1-\sin\theta}$ .

**Solution:**  $-\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}} + C.$

17.

$$\int \frac{x}{\sqrt{1-2x^2-x^4}} dx$$

**Solution:**  $\frac{1}{2} \arcsin \frac{x^2+1}{\sqrt{2}} + C$

18.

$$\int \frac{x+1}{\sqrt{(x^2+1)^3}} dx$$

**Solution:**  $\frac{x-1}{\sqrt{x^2+1}} + C.$

19.

$$\int \sqrt{x^2 - 9} \, dx$$

**Solution:**  $\frac{x}{2}\sqrt{x^2 - 9} - \frac{9}{2} \ln |x + \sqrt{x^2 - 9}| + C$

20.

$$\int \sqrt{x - 4x^2} \, dx$$

**Solution:**  $\frac{1}{16}(8x - 1)\sqrt{x - 4x^2} + \frac{1}{64} \arcsin(8x - 1) + C$

21.

$$\int x\sqrt{x^2 + 2x + 2} \, dx$$

**Solution:**  $\frac{1}{3}(x^2 + 2x + 2)^{\frac{3}{2}} - \frac{x+1}{2}\sqrt{x^2 + 2x + 2} - \frac{1}{2} \ln(x + 1 + \sqrt{x^2 + 2x + 2}) + C.$

22.

$$\int \frac{1}{x^4\sqrt{x^2 - 1}} \, dx$$

**Solution:**  $\frac{\sqrt{x^2-1}}{x} - \frac{(x^2-1)^{\frac{3}{2}}}{3x^3} + C.$

23.

$$\int \cos^4 x \, dx$$

**Solution:**  $\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$

24.

$$\int \frac{1}{\cos x \sin^5 x} \, dx$$

**Solution:**  $\ln |\tan x| - \cot^2 x - \frac{1}{4} \cot^4 x + C.$

25.

$$\int \frac{1 + \sqrt{\cot x}}{\sin^2 x} dx$$

**Solution:**  $-\cot x - \frac{2\sqrt{\cot^3 x}}{3} + C.$

26.

$$\int \frac{\sin^3 x}{\sqrt[5]{\cos^3 x}} dx$$

**Solution:**  $\frac{5}{12}(\cos^2 x - 6)\sqrt[5]{\cos^2 x} + C.$

27.

$$\int \frac{1}{\sin^3(5x)} dx$$

**Solution:**  $\frac{1}{10} \frac{\cos 5x}{\sin^2 5x} - \frac{1}{20} \ln \frac{1+\cos 5x}{1-\cos 5x} + C$

28.

$$\int \frac{\sin^2 x}{\cos^6 x} dx$$

**Solution:**  $\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

29.

$$\int \frac{1}{\cos^2 x \sqrt{\tan^2 x + 4 \tan x + 1}} dx$$

**Solution:**  $\ln |\tan x + 2 + \sqrt{\tan^2 x + 4 \tan x + 1}| + C$

30.

$$\int \frac{x}{\cos^2 3x} dx$$

**Solution:**  $\frac{1}{3}x \tan 3x + \frac{1}{9} \ln |\cos 3x| + C.$

31.

$$\int x \sin^2 x \, dx$$

**Solution:**  $\frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8} + C.$

32.

$$\int x^2 e^{x^3} \, dx$$

**Solution:**  $\frac{1}{3}e^{x^3} + C$

33.

$$\int x e^{2x} \, dx$$

**Solution:**  $\frac{e^{2x}}{4}(2x - 1) + C.$

34.

$$\int x \ln \sqrt{x-1} \, dx$$

**Solution:**  $\frac{x^2-1}{4} \ln |x-1| - \frac{x}{4} - \frac{x^2}{8} + C.$

35.

$$\int \frac{x \arctan x}{\sqrt{x^2+1}} \, dx$$

**Solution:**  $\sqrt{1+x^2} \arctan x - \ln(x + \sqrt{1+x^2}) + C.$

36.

$$\int \frac{1}{(\cos x + \sin x)^2} \, dx$$

*Hint.* Factor out  $\cos x$ .

**Solution:**  $-\frac{1}{1+\tan x} + C$

37.

$$\int \frac{1}{(\tan x + 1) \sin^2 x} \, dx$$

**Solution:**  $\ln |1 + \cot x| - \cot x + C$

38.

$$\int \frac{1}{e^{2x} - 2e^x} \, dx$$

**Solution:**  $\frac{1}{2e^x} - \frac{x}{4} + \frac{1}{4} \ln |e^x - 2| + C$

39.

$$\int \frac{e^x}{e^{2x} - 6e^x + 13} \, dx$$

**Solution:**  $\frac{1}{2} \arctan \frac{e^x - 3}{2} + C$

40.

$$\int \frac{e^{2x}}{(e^x + 1)^{\frac{1}{4}}} \, dx$$

**Solution:**  $\frac{4}{7} \sqrt[4]{(e^x + 1)^7} - \frac{4}{3} \sqrt[4]{(e^x + 1)^3} + C$

41.

$$\int \frac{2^x}{1 - 4^x} \, dx$$

**Solution:**  $\frac{1}{\ln 4} \ln \frac{1+2^x}{1-2^x} + C.$

42.

$$\int (x^2 - 1)e^{2x} \, dx$$

**Solution:**  $-\frac{10^{-2x}}{2\ln 10} \left(x^2 - 1 + \frac{x}{\ln 10} + \frac{1}{2\ln^2 10}\right) + C.$

43.

$$\int \sqrt{e^x + 1} \, dx$$

**Solution:**  $2\sqrt{e^x + 1} + \ln \frac{\sqrt{e^x + 1} - 1}{\sqrt{e^x + 1} + 1} + C$

44.

$$\int \frac{\arctan x}{x^2} \, dx$$

**Solution:**  $\ln \left| \frac{x}{\sqrt{x^2 + 1}} \right| - \frac{\arctan x}{x} + C.$

45.

$$\int x^3 \arcsin \frac{1}{x} \, dx$$

**Solution:**  $\frac{1}{4} \left( x^4 \arcsin \frac{1}{x} + \frac{x^2 + 2}{3} \sqrt{x^2 - 1} \right) + C.$

46.

$$\int \cos(\ln x) \, dx$$

**Solution:**  $\frac{x}{2} (\cos(\ln x) + \sin(\ln x)) + C.$

47.

$$\int (x^2 - 3x) \sin 5x \, dx$$

**Solution:**  $\frac{1}{5} \left( -x^2 \cos 5x + \frac{2}{5} x \sin 5x + 3x \cos 5x + \frac{2}{25} \cos 5x - \frac{3}{5} \sin 5x \right) + C.$

48.

$$\int x \arctan(2x + 3) \, dx$$

**Solution:**  $\frac{1}{2} \left[ (x^2 - 2) \arctan(3x + 3) + \frac{3}{4} \ln(2x^2 + 6x + 5) - \frac{x}{2} \right] + C.$

49.

$$\int \arcsin \sqrt{x} \, dx$$

**Solution:**  $\frac{1}{2} \sqrt{x - x^2} + \left(x - \frac{1}{2}\right) \arcsin \sqrt{x} + C$

50. Find the arc length of the parabola  $y^2 = 4x$  from  $x = 0$  to  $x = 1$ .

**Solution:**  $\sqrt{2} + \ln(1 + \sqrt{2}).$

51. Find the arc length of the curve  $y = \ln x$  from  $x = \sqrt{3}$  to  $x = \sqrt{8}$ .

**Solution:**  $1 + \frac{1}{2} \ln \frac{3}{2}$

52. Find the arc length of the curve  $y = \arcsin e^{-x}$  from  $x = 0$  to  $x = 1$ .

**Solution:**  $\ln(e + \sqrt{e^2 - 1})$

53. Find the arc length of the curve  $x = \frac{1}{4}y^2 - \frac{1}{2} \ln y$  from  $y = 1$  to  $y = e$ .

**Solution:**  $\frac{1}{4}(e^2 + 1)$

54. Find the area of a surface formed by the rotation, about  $x$ -axis or an arc of the curve  $9x = y^2 + 18$ , from  $x = 2$  to  $x = 6$ .

**Solution:**  $49\pi$

55. Find the area of a surface formed by the rotation, about  $x$ -axis or an arc of the curve  $y = \frac{x^3}{6} + \frac{1}{2x}$ , from  $x = \frac{1}{2}$  to  $x = 1$ .

**Solution:**  $\frac{263}{256}\pi.$

56. Find the area of a surface formed by the rotation, about  $y$ -axis or an arc of the curve  $x = \sqrt{1 - y}$ , from  $x = 0$  to  $x = 1$ .

**Solution:**  $\frac{\pi}{6}(5\sqrt{5} - 1)$