

Review 1.

1. Determine, whether the function $\arctan(e^x)$ defined on $(-\infty, \infty)$ is one to one.
2. Determine, whether the function $\ln(x^2 + 1)$ defined on $(-\infty, \infty)$ is one to one.
3. Determine, whether the function

$$f(x) = \begin{cases} x & x \leq 0, \\ x^2 & x > 0, \end{cases}$$

is one to one.

4. Based on a graph, you should be able to determine whether a function is one to one.
5. Given a function f with $f(0) = 1$, $f(1) = 3$, $f(2) = 0$, $f(4) = -3$. Determine $f^{-1}(0)$.
6. What is the domain of f^{-1} if $f(x) = e^x + 1$?
7. What is the domain of f^{-1} if $f(x) = \frac{1}{x}$?
8. Find a formula for the inverse of the function $f(x) = 2x^2 - 8x$, $x \leq 2$.
9. Find a formula for the inverse of the function $f(x) = \frac{4e^x - 1}{2e^x + 3}$.
10. Find a formula for the inverse of the function $f(x) = \sqrt{10 - 3\sin(2x)}$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.
11. Find a formula for the inverse of the function $f(x) = \arctan(\sqrt[3]{x^2 + 1})$, $x \leq 0$.
12. Verify that $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$, that is, verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
13. Find the expression for the inverse of the functions of $[f(x)]^2$ and $f(x^2)$, $x \geq 0$, in terms of f^{-1} .
14. Make rough sketch of the graph of the function $y = \frac{1}{2^{x-3}}$.
15. Make rough sketch of the graph of the function $y = 1 - 4^x$.
16. Find C such that for the function $f(x) = Ce^x$ one has $f(2) = e^{-3}$.
17. Find the limit $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^x}{e^{2x} + e^x}$.
18. Find the limit $\lim_{x \rightarrow -\infty} \frac{e^x + \sin(x) + 2}{e^x - \sin(x) - 2}$.
19. Find the limit $\lim_{x \rightarrow \infty} \frac{2^x + 3^x}{5^x}$.
20. Find the limit $\lim_{x \rightarrow \infty} \frac{2e^{2x} - 3e^{3x}}{e^{x^2}}$.
21. Differentiate the function $\sin(e^x) + e^{-x^2}$.
22. Differentiate the function $\sqrt{1 + xe^{-2x}}$.
23. Evaluate the integral

$$\int \frac{(1 + e^x)^2}{e^x} dx$$

24. Evaluate the integral

$$\int_0^{\infty} e^{-x} \cos(e^{-x}) dx$$

25. Find the value of x such that $e^{2x} + e^x = 6$.

26. Find the exact value of $e^{\ln 15}$.

27. Find the exact value of $\log_8 320 - \log_8 5$.

28. Sketch the graph of $\ln(x + 2)$.

29. Sketch the graph of $\log_{\frac{1}{2}} x^2$.

30. Solve the equation $\ln x + \ln(x - 1) = 1$ for x .

31. Solve the equation $\log_3 x + \ln(x^3) = 1$ for x .

32. Find the limit $\lim_{x \rightarrow 0^+} \ln(\sin x)$.

33. Find the limit $\lim_{x \rightarrow \infty} \frac{\ln(x^3 e^{-x})}{x + \ln x}$

34. Find the limit $\lim_{x \rightarrow \infty} \ln(x^2 + 2x) - \ln(2x^2 - 2x)$

35. Find the limit $\lim_{x \rightarrow \infty} \log_3 x - \log_2 x$

36. Differentiate the function $y \ln(1 + e^y)$

37. Differentiate the function e^{2^x}

38. Differentiate the function $\sqrt{x}e^{x^2}(x^2 + 1)^{10}$.

39. Differentiate the function $(\sin x)^{\ln x}$.

40. Evaluate the integral

$$\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx.$$

41. Evaluate the integral

$$\int \frac{\cos x}{2 + \sin x} dx$$

42. Find the exact value of $\arccos(-\frac{1}{2})$.

43. Find the exact value of $\arcsin(\sin \frac{7\pi}{3})$.

44. Simplify the expression $\sin(\arctan x)$.

45. Find the derivative of the function $\arctan \sqrt{\frac{1-x}{1+x}}$.

46. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

47. Evaluate the integral

$$\int \frac{e^{2x}}{\sqrt{1 - e^{4x}}} dx$$

48. Evaluate the integral

$$\int_2^3 \frac{\ln[x^2]}{x} dx$$

49. Evaluate the integral and simplify the answer such that there is no trigonometric, or inverse trigonometric function

$$\int \frac{\sin(\arctan x)}{1+x^2} dx.$$

50. Find the numerical value for $\cosh(\ln 3)$.

51. Prove the identity

$$\sinh^2\left(\frac{x}{2}\right) = \frac{1}{2}(\cosh(x) - 1).$$

52. Show that $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$.

53. If $\tanh x = \frac{12}{13}$, find $\cosh x$.

54. Find the derivative of $x \sinh x - \cosh x^2$.

55. Evaluate the integral

$$\int \tanh x dx$$

56. Find the limit

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3}$$

57. Find the limit

$$\lim_{x \rightarrow -\infty} x^2 e^x$$

58. Find the limit

$$\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x} \right)$$

59. Find the limit

$$\lim_{x \rightarrow \infty} (xe^{\frac{1}{x}} - x)$$

60. Find the limit

$$\lim_{x \rightarrow \infty} x^{\frac{1}{\ln x}}$$

61. Find the limit

$$\lim_{x \rightarrow 1} (2-x)^{\tan(\pi x/2)}$$

62. Find the limit

$$\lim_{x \rightarrow 0} \frac{\cos x}{1-x}$$

63. Find the limit

$$\lim_{x \rightarrow \infty} x^x$$

64. Find the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^{\ln x}$$

65. Find the limit

$$\lim_{x \rightarrow \infty} (\arctan x)^x$$

66. Find the limit

$$\lim_{x \rightarrow \infty} \left(\frac{1}{1+x^2} \right)^{e^x}$$

67. Find the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^{\tan x}$$