

# Review 1.

1. Determine, whether the function  $\arctan(e^x)$  defined on  $(-\infty, \infty)$  is one to one.

**Solution:** Yes.

2. Determine, whether the function  $\ln(x^2 + 1)$  defined on  $(-\infty, \infty)$  is one to one.

**Solution:** No.

3. Determine, whether the function

$$f(x) = \begin{cases} x & x \leq 0, \\ x^2 & x > 0, \end{cases}$$

is one to one.

**Solution:** Yes.

4. Based on a graph, you should be able to determine whether a function is one to one.

**Solution:** No answer.

5. Given a function  $f$  with  $f(0) = 1$ ,  $f(1) = 3$ ,  $f(2) = 0$ ,  $f(4) = -3$ . Determine  $f^{-1}(0)$ .

**Solution:** 2

6. What is the domain of  $f^{-1}$  if  $f(x) = e^x + 1$ ?

**Solution:** Domain is  $(1, \infty)$

7. What is the domain of  $f^{-1}$  if  $f(x) = \frac{1}{x}$ ?

**Solution:**  $(-\infty, 0) \cup (0, \infty)$ .

8. Find a formula for the inverse of the function  $f(x) = 2x^2 - 8x$ ,  $x \leq 2$ .

**Solution:**  $2 - \sqrt{4 + \frac{y}{2}}$

9. Find a formula for the inverse of the function  $f(x) = \frac{4e^x - 1}{2e^x + 3}$ .

**Solution:**  $\ln\left(\frac{3y+1}{4-2y}\right)$ .

10. Find a formula for the inverse of the function  $f(x) = \sqrt{10 - 3\sin(2x)}$ ,  $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ .

**Solution:**  $\frac{1}{2} \arcsin\left(\frac{10-y^2}{3}\right)$ .

11. Find a formula for the inverse of the function  $f(x) = \arctan(\sqrt[3]{x^2 + 1})$ ,  $x \leq 0$ .

**Solution:**  $-\sqrt{(\tan(y))^3 - 1}$

12. Verify that  $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$ , that is, verify that  $f(f^{-1}(x)) = x$  and  $f^{-1}(f(x)) = x$ .

**Solution:** No answer.

13. Find the expression for the inverse of the functions of  $[f(x)]^2$  and  $f(x^2)$ ,  $x \geq 0$ , in terms of  $f^{-1}$ .

**Solution:**  $[f^2]^{-1}(x) = f^{-1}(\sqrt{x})$ ,  $[f(x^2)]^{-1} = \sqrt{f^{-1}(x)}$ .

14. Make rough sketch of the graph of the function  $y = \frac{1}{2^{x-3}}$ .

**Solution:** No answer.

15. Make rough sketch of the graph of the function  $y = 1 - 4^x$ .

**Solution:** No answer.

16. Find  $C$  such that for the function  $f(x) = Ce^x$  one has  $f(2) = e^{-3}$ .

**Solution:**  $C = e^{-5}$ .

17. Find the limit  $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^x}{e^{2x} + e^x}$ .

**Solution:**  $\infty$

18. Find the limit  $\lim_{x \rightarrow -\infty} \frac{e^x + \sin(x) + 2}{e^x - \sin(x) - 2}$

**Solution:**  $-1$ .

19. Find the limit  $\lim_{x \rightarrow \infty} \frac{2^x + 3^x}{5^x}$ .

**Solution:**  $0$ .

20. Find the limit  $\lim_{x \rightarrow \infty} \frac{2e^{2x} - 3e^{3x}}{e^{x^2}}$ .

**Solution:**  $0$

21. Differentiate the function  $\sin(e^x) + e^{-x^2}$

**Solution:**  $\cos(e^x)e^x - 2xe^{-x^2}$ .

22. Differentiate the function  $\sqrt{1 + xe^{-2x}}$ .

**Solution:**  $\frac{1}{2\sqrt{1+xe^{-2x}}}(e^{-2x} - 2xe^{-2x})$ .

23. Evaluate the integral

$$\int \frac{(1 + e^x)^2}{e^x} dx$$

**Solution:**  $-e^{-x} + 2x + e^x + C$

24. Evaluate the integral

$$\int_0^\infty e^{-x} \cos(e^{-x}) dx$$

**Solution:**  $\sin(1)$ .

25. Find the value of  $x$  such that  $e^{2x} + e^x = 6$ .

**Solution:**  $x = \ln 2$ .

26. Find the exact value of  $e^{\ln 15}$ .

**Solution:** 15.

27. Find the exact value of  $\log_8 320 - \log_8 5$ .

**Solution:** 2

28. Sketch the graph of  $\ln(x+2)$ .

**Solution:** No answer.

29. Sketch the graph of  $\log_{\frac{1}{2}} x^2$ .

**Solution:** No answer.

30. Solve the equation  $\ln x + \ln(x-1) = 1$  for  $x$ .

**Solution:**  $x = \frac{1+\sqrt{1+4e}}{2}$ .

31. Solve the equation  $\log_3 x + \ln(x^3) = 1$  for  $x$ .

**Solution:**  $x = e^{\frac{\ln 3}{1+3\ln 3}}$ .

32. Find the limit  $\lim_{x \rightarrow 0^+} \ln(\sin x)$ .

**Solution:**  $-\infty$ .

33. Find the limit  $\lim_{x \rightarrow \infty} \frac{\ln(x^3 e^{-x})}{x + \ln x}$

**Solution:** -1.

34. Find the limit  $\lim_{x \rightarrow \infty} \ln(x^2 + 2x) - \ln(2x^2 - 2x)$

**Solution:**  $-\ln 2$ .

35. Find the limit  $\lim_{x \rightarrow \infty} \log_3 x - \log_2 x$

**Solution:**  $-\infty$ .

36. Differentiate the function  $y \ln(1 + e^y)$

**Solution:**  $\ln(1 + e^y) + \frac{ye^y}{1+e^y}$ .

37. Differentiate the function  $e^{2^x}$

**Solution:**  $e^{2^x} 2^x \ln 2$

38. Differentiate the function  $\sqrt{x}e^{x^2}(x^2 + 1)^{10}$ .

**Solution:** By logarithmic differentiation:  $\sqrt{x}e^{x^2}(x^2 + 1)^{10} \left( \frac{1}{2x} + 2x + \frac{20x}{x^2+1} \right)$ .

39. Differentiate the function  $(\sin x)^{\ln x}$ .

**Solution:** By logarithmic differentiation:  $(\sin x)^{\ln x} \left( \frac{\ln(\sin x)}{x} + \ln x \frac{\cos x}{\sin x} \right)$ .

40. Evaluate the integral

$$\int_4^9 \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx.$$

**Solution:**  $\frac{85}{2} + \ln 9 - \ln 4$ .

41. Evaluate the integral

$$\int \frac{\cos x}{2 + \sin x} dx$$

**Solution:**  $\ln |2 + \sin x| + C$ .

42. Find the exact value of  $\arccos(-\frac{1}{2})$ .

**Solution:**  $\frac{2\pi}{3}$ .

43. Find the exact value of  $\arcsin(\sin \frac{7\pi}{3})$ .

**Solution:**  $\frac{\pi}{3}$ .

44. Simplify the expression  $\sin(\arctan x)$ .

**Solution:**  $\frac{x}{\sqrt{1+x^2}}$ .

45. Find the derivative of the function  $\arctan \sqrt{\frac{1-x}{1+x}}$ .

**Solution:**  $\frac{-1}{2\sqrt{1-x^2}}$ .

46. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

**Solution:**  $\frac{\pi}{4}$ .

47. Evaluate the integral

$$\int \frac{e^{2x}}{\sqrt{1 - e^{4x}}} dx$$

**Solution:**  $\frac{1}{2} \sin^{-1}(e^{2x}) + C$ .

48. Evaluate the integral

$$\int_2^3 \frac{\ln[x^2]}{x} dx$$

**Solution:**  $(\ln 3)^2 - (\ln 2)^2$ .

49. Evaluate the integral and simply the answer such that there is no trigonometric, or inverse trigonometric function

$$\int \frac{\sin(\arctan x)}{1 + x^2} dx .$$

**Solution:**  $-\frac{1}{\sqrt{1+x^2}} + C.$

50. Find the numerical value for  $\cosh(\ln 3)$ .

**Solution:**  $\frac{5}{3}.$

51. Prove the identity

$$\sinh^2\left(\frac{x}{2}\right) = \frac{1}{2}(\cosh(x) - 1).$$

**Solution:** No answer.

52. Show that  $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right).$

**Solution:** No answer.

53. If  $\tanh x = \frac{12}{13}$ , find  $\cosh x$ .

**Solution:**  $\frac{13}{5}.$

54. Find the derivative of  $x \sinh x - \cosh x^2$ .

**Solution:**  $\sinh x + x \cosh x - 2x \sinh(x^2).$

55. Evaluate the integral

$$\int \tanh x \, dx$$

**Solution:**  $\ln |\cosh x| + C.$

56. Find the limit

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3}$$

**Solution:**  $\frac{1}{6}.$

57. Find the limit

$$\lim_{x \rightarrow -\infty} x^2 e^x$$

**Solution:** 0.

58. Find the limit

$$\lim_{x \rightarrow 0} \left( \cot x - \frac{1}{x} \right)$$

**Solution:** 0.

59. Find the limit

$$\lim_{x \rightarrow \infty} (x e^{\frac{1}{x}} - x)$$

**Solution:** 1.

60. Find the limit

$$\lim_{x \rightarrow \infty} x^{\frac{1}{\ln x}}$$

**Solution:**  $e$ .

61. Find the limit

$$\lim_{x \rightarrow 1} (2 - x)^{\tan(\pi x/2)}$$

**Solution:**  $e^{2/\pi}$ .

62. Find the limit

$$\lim_{x \rightarrow 0} \frac{\cos x}{1 - x}$$

**Solution:** 1.

63. Find the limit

$$\lim_{x \rightarrow \infty} x^x$$

**Solution:**  $\infty$ .

64. Find the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^{\ln x}$$

**Solution:**  $\infty$ .

65. Find the limit

$$\lim_{x \rightarrow \infty} (\arctan x)^x$$

**Solution:**  $\infty$ .

66. Find the limit

$$\lim_{x \rightarrow \infty} \left( \frac{1}{1+x^2} \right)^{e^x}$$

**Solution:** 0.

67. Find the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^{\tan x}$$

**Solution:** 1.