Test 1 answer key.

(4 points)

Yellow version: What is the domain of the function $\sqrt[4]{x-7}$?

$x \geq 7$ or $[\,7,\,\infty)$

White version: What is the domain of the function $\sqrt{x^3-7}$?

$x \geq \sqrt[3]{7}$ or $[\,\sqrt[3]{7},\,\infty)$

Partial credit: I took off one point if the interval was written as $x > 7$ instead of $x \geq 7$, or if it was written as $(7, \infty)$ instead of $[7, \infty)$. Similarly for $\sqrt[3]{7}$.

The number $\sqrt[3]{7}$ is approximately equal to 1.9129; I gave full credit if $\sqrt[3]{7}$ was replaced with 1.9. I took off a point if it was replaced by 2.

Some students seemed to think that the answer was $x \neq 7$ (or, respectively, $x \neq \sqrt[3]{7}$). I don't know how they came up with that answer; I gave 0 points for it.

(4 points)

Yellow version: Let $p(x) = 2x + 1$ and $q(x) = x^2 - 1$. Then $p(q(4)) =$ 31 and $q(p(4)) =$ 80.

Intermediate steps (optional): $p(4) = 9$ and $q(4) = 15$.

White version: Let $f(x) = x^2 + 1$ and $g(x) = 2x - 1$. Then $f(g(3)) =$ 26 and $g(f(3)) =$ 19.

Intermediate steps (optional): $f(3) = 10$ and $g(3) = 5$.

Each of the two numbers was worth 2 points, when no other method of evaluation seemed evident.

I deducted 1 point for switching the two numbers.

I took off 1 point for what appeared to be a simple arithmetic error – e.g., when a student wrote 31 and 79 instead of 31 and 80.

I took off 2 points for conceptual errors that seemed to result from not knowing the distributive law. For instance, some students seemed to think that $q(p(4)) = (2 \cdot 4 + 1)^2 - 1$ is equal to $(2 \cdot 4^2 + 1^2) - 1 = 32$. Similarly, some students seemed to think that $q(p(4)) = (2 \cdot 4 + 1)^2 - 1$ is equal to $[(2 \cdot 4)^2 + 1^2] - 1 = 64$.

(4 points)

Yellow version: $\lim_{x \to -1} \frac{x^3 + 2x + 1}{x + 3} = \boxed{-1}$

Intermediate steps (optional): You can just plug in $x = -1$, since this in this problem that doesn't yield 0 in the denominator. Thus $\frac{(-1)^3 + 2(-1) + 1}{(-1) + 3} = \frac{-1 - 2 + 1}{-1 + 3} = \frac{-2}{2} = \boxed{-1}$

White version: $\lim_{x \to 2} \frac{x^3 + 2x + 1}{x + 3} = \boxed{13/5 \text{ or } 2.6}$

Intermediate steps (optional): $\frac{2^3 + 2 \cdot 2 + 1}{2 + 3}$

Partial credit: Only one point was deducted if the right method was used (i.e., plugging in -1 or 2 for x) but an arithmetic error happened en route.

Zero credit was given for using an entirely wrong method. For instance, factoring out the highest power of x is not helpful in this problem, since we're not taking the limit as $x \to \infty$.

(5 points)

Yellow version: $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} = \boxed{1/2 \text{ or } .5}$

Intermediate steps (optional):

$$\frac{1-\sqrt{x}}{1-x} = \frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})} = \frac{1}{1+\sqrt{x}} \rightarrow \frac{1}{1+\sqrt{1}}$$

White version: $\lim_{x\to\sqrt{2}} \frac{x-\sqrt{2}}{x^2-2} =$ $\boxed{\frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}}$

Intermediate steps (optional):

$$\frac{x-\sqrt{2}}{(x-\sqrt{2})(x+\sqrt{2})} = \frac{1}{x+\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}+\sqrt{2}}$$

Partial credit:

Deduct 1 point for insufficient simplification, e.g., $\frac{1}{\sqrt{2}+\sqrt{2}}$

Deduct 1 point for minor transcription error (i.e., copying something wrong from one place to another). Deduct 2 points for a major transcription error – e.g., one student worked out the correct answer but nevertheless wrote "DNE" in the answer box.

Full credit for an answer of 0.353 or 0.354 in the white version.

Deduct 4 points (i.e., give 1 point) for simply writing 0/0 (recognizing it's that type of problem is a good start).

Deduct 3 points (i.e., give 2 points) for factoring correctly — i.e., for noting $1-x=(1-\sqrt{x})(1+\sqrt{x})$ or $x^2-2=(x-\sqrt{2})(x+sqrt2)$.

Deduct 4 points (i.e., give 1 point) for noting that $\frac{1-\sqrt{x}}{1-x} = \frac{1-\sqrt{x}}{1-x}\cdot\frac{1+\sqrt{x}}{1+\sqrt{x}}$. That's correct, but you'd have to carry it a little farther to make it useful. You'd continue with $= \frac{1-x}{(1-x)(1+\sqrt{x})}$ (worth 2 points) $= \frac{1}{1+\sqrt{x}}$ (worth 3 points), and then finally take limits.

(5 points)

Yellow version: $\lim_{x\to\infty} \frac{3x^2+7x+1}{5x^2-5x+1} =$ $\boxed{3/5}$

Intermediate steps (optional): the exponents match, so use the leading coefficients. Or,

$$\lim_{x\to\infty} \frac{3x^2+7x+1}{5x^2-5x+1} = \lim_{x\to\infty} \frac{x^2\left(3+\frac{7}{x}+\frac{1}{x^2}\right)}{x^2\left(5-\frac{5}{x}+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{3+\frac{7}{x}+\frac{1}{x^2}}{5-\frac{5}{x}+\frac{1}{x^2}} = \frac{3+0+0}{5-0+0} = \boxed{\frac{3}{5}}.$$

White version: $\lim_{x\to\infty} \frac{x^3+7x^2+2x+1}{4x^3+6x-2} = \boxed{1/4}$

Intermediate steps (optional): the exponents match, so use the leading coefficients. Or, $\lim_{x\to\infty} \frac{x^3+7x^2+2x+1}{4x^3+6x-2} = \lim_{x\to\infty} \frac{x^3\left(1+\frac{7}{x}+\frac{2}{x^2}+\frac{1}{x^3}\right)}{x^3\left(4+\frac{6}{x^2}-\frac{2}{x^3}\right)} = \lim_{x\to\infty} \frac{1+\frac{7}{x}+\frac{2}{x^2}+\frac{1}{x^3}}{4+\frac{6}{x^2}-\frac{2}{x^3}} = \frac{1+0+0+0}{4+0-0} = \boxed{\frac{1}{4}}$

Partial credit: A wrong answer accompanied by incomprehensible work generally was worth 0 or 1 points.

One common error was, in dividing all the terms through by x^2 or by x^3, one term was overlooked; this was judged as a transcription error and so it only cost 1 point. However, if more than one term was overlooked, this was judged as a conceptual error, and so 4 points were deducted.

(4 points)

Yellow version: $\lim_{x\to\infty} \frac{x^3+7x^2+2x+1}{4x^2+6x-2} = \boxed{\text{DNE}}$

Intermediate steps (optional): deg(numerator) > deg(denominator).

Or, in more detail: $\lim_{x\to\infty} \frac{x^3+7x^2+2x+1}{4x^2+6x-2} = \lim_{x\to\infty} \frac{x^3\left(1+\frac{7}{x}+\frac{2}{x^2}+\frac{1}{x^3}\right)}{x^3\left(\frac{4}{x}+\frac{6}{x^2}-\frac{2}{x^3}\right)} = \lim_{x\to\infty} \frac{1+\frac{7}{x}+\frac{2}{x^2}+\frac{1}{x^3}}{\frac{4}{x}+\frac{6}{x^2}-\frac{2}{x^3}}$

$= \lim \frac{\text{something getting near 1}}{\text{something getting near 0}} = \text{limit does not exist}$

White version: $\lim_{x\to\infty} \frac{3x^2+7x+1}{5x^3-5x+1} = \boxed{0}$

Intermediate steps (optional):
deg(numerator) < deg(denominator). Or, in more detail:

$$\lim_{x\to\infty} \frac{3x^2+7x+1}{5x^3-5x+1} = \lim_{x\to\infty} \frac{x^3\left(\frac{3}{x}+\frac{7}{x^2}+\frac{1}{x^3}\right)}{x^3\left(5-\frac{5}{x^2}+\frac{1}{x^3}\right)} = \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{7}{x^2}+\frac{1}{x^3}}{5-\frac{5}{x^2}+\frac{1}{x^3}} =$$
$$= \lim \frac{\text{something getting near } 0}{\text{something getting near } 5} = 0$$

Partial credit:

Give 1 point for dividing by the wrong power of x, if this leads to a wrong answer.

Give 2 points if dividing by the right power of x but ignoring one of the resulting terms or factors.

(4 points)

Yellow version: Let $g(x) = \begin{cases} x-3 & (x<2) \\ 2 & (x=2) \\ x^2+1 & (x>2) \end{cases}$ Find (if they exist):

$\lim_{x\to 2-} g(x) =$ -1 $\quad \lim_{x\to 2+} g(x) =$ 5 $\quad \lim_{x\to 2} g(x) =$ DNE $\quad g(2) =$ 2

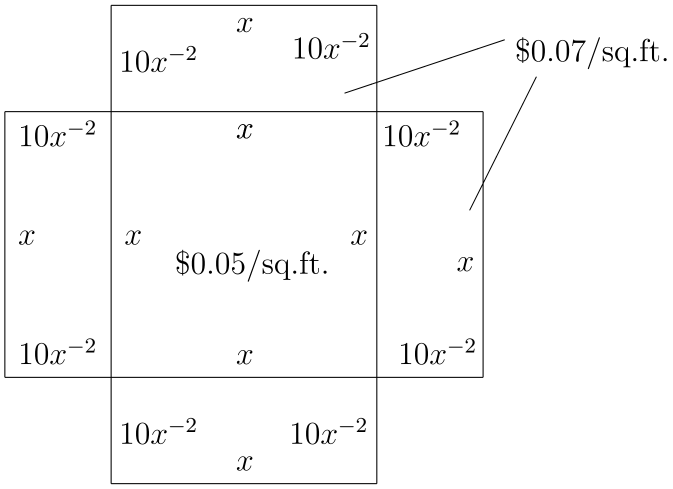
White version: Let $f(x) = \begin{cases} x^3-1 & (x<1) \\ 3 & (x=1) \\ x+1 & (x>1) \end{cases}$ Find (if they exist):

$\lim_{x\to 1-} f(x) =$ 0 $\quad \lim_{x\to 1+} f(x) =$ 2 $\quad \lim_{x\to 1} f(x) =$ DNE $\quad f(1) =$ 3

Partial credit: One point for each of the four questions; no fractions of points; each question must be answered correctly to get the point.

(5 points) (both versions of test)

A box is to be made with a square bottom, x feet by x feet. The desired volume of the box is 10 cubic feet; therefore the height of the box must be $10x^{-2}$ feet. The square bottom of the box is made of material costing 5 cents per square foot, and the four rectangular sides of the box are made of material costing 7 cents per square foot. The box will have no top. Express the cost of the box as a function of x.



Answer. (Same problem on both tests.)

	sides	bottom
area	$4 \cdot 10x^{-1}$	x^2
cost/sq-ft	\$0.07	\$0.05
total cost	$\$2.80x^{-1}$	$\$0.05x^2$

Adding, we get $\boxed{\$2.80x^{-1} + \$0.05x^2}$ or $\boxed{\$\frac{14}{5}x^{-1} + \$\frac{1}{20}x^2}$. (No penalty for omitting the dollar signs.)

Deductions for partial credit:

Deduct 2 points for miscalculating a decimal or losing a factor.

Deduct 4 points if answer was wrong but method of work was somewhat evident and understandable.

Deduct 2 points if method was right but there were several arithmetic mistakes.

Deduct 1 point if there was one algebraic mistake.

Deduct 1 point if the student gave the right answer and then plugged in some number for x.