Quiz 2 answers

(4 points)

White version: Find the line tangent to $y = x^3 + x$ at $(1, 2)$.

Solution. $y' = 3x^2 + 1$, so when $x = 1$ we get $y' = 4$. Thus we want a line through $(1, 2)$ with slope 4. That is $\boxed{y - 2 = 4(x - 1)}$ or $\boxed{y = 4x - 2}$.

Green version: Find the line tangent to $y = 2\sqrt{x} + 1$ at $(1, 3)$.

Solution. $y = 2x^{1/2} + 1$, so $y' = x^{-1/2}$, and when $x = 1$ then $y' = 1$. Thus we want a line through $(1, 3)$ with slope 1. That is $\boxed{y - 3 = x - 1}$ or $\boxed{y = x + 2}$.

Partial credit: I gave one point for finding the formula for dy/dx (a function of x), or two points if you found that formula and then plugged in $x = 1$ to find the slope of the tangent line. I also gave two points if you found an equation for a line that passed through the given point.

Find the derivative of each of the following functions:

(3 points)

Green version: $a(x) = 3x^7 + 5x^{1/2} - 2x^{-1/3}$

Solution.

$$a'(x) = \boxed{21x^6 + \tfrac{5}{2}x^{-1/2} + \tfrac{2}{3}x^{-4/3}} = \boxed{21x^6 + 2.5x^{-1/2} + \tfrac{2}{3}x^{-4/3}}.$$

Some students felt it was important to avoid using negative exponents (I'm not sure why); that yields the answer

$$\boxed{21x^6 + \frac{5}{2x^{1/2}} + \frac{2}{3x^{4/3}}} = \boxed{21x^6 + \frac{5}{2\sqrt{x}} + \frac{2}{3\sqrt[3]{x^4}}}.$$

White version: $b(x) = 2x^5 - 3x^{1/3} + 2x^{-1/5}$

Solution.

$$b'(x) = \boxed{10x^4 - x^{-2/3} - \tfrac{2}{5}x^{-6/5}} = \boxed{10x^4 - x^{-2/3} - 0.4x^{-6/5}}.$$

Again, it is also possible the express the answer without negative exponents:

$$\boxed{10x^4 - \frac{1}{x^{2/3}} - \frac{2}{5x^{6/5}}} = \boxed{10x^4 - \frac{1}{\sqrt[3]{x^2}} - \frac{2}{5\sqrt[6]{x^5}}}.$$

Partial credit: The answer is a sum of three terms; each term was worth one point. Be careful about $\pm$ signs.

(4 points)

White version: $f(x) = \dfrac{x^2+1}{x+3}$

Solution: Use the quotient rule, $\left(\frac{p}{q}\right)' = \frac{p'q-pq'}{q^2}$, in this case with $p(x) = x^2+1$ and $q(x) = x+3$. We obtain $p' = 2x$ and $q' = 1$, so

$$\frac{p'q-pq'}{q^2} = \frac{(2x)(x+3)-(x^2+1)(1)}{(x+3)^2} = \boxed{\frac{x^2+6x-1}{(x+3)^2}}$$

Full credit will also be given if the denominator is written out as $x^2 + 6x + 9$.

Green version: $g(x) = \dfrac{x+1}{x^2+3}$

Solution: Use the quotient rule, $\left(\frac{p}{q}\right)' = \frac{p'q-pq'}{q^2}$, in this case with $p(x) = x+1$ and $q(x) = x^2+3$. Then $p' = 1$ and $q' = 2x$, so

$$\frac{p'q-pq'}{q^2} = \frac{(1)(x^2+3)-(x+1)(2x)}{(x^2+3)^2} = \boxed{\frac{-x^2-2x+3}{(x^2+3)^2}}$$

or $\boxed{-\dfrac{x^2+2x-3}{(x^2+3)^2}}$. Full credit will also be given if the denominator is written out as x^4+6x^2+9.

Partial credit for assorted common errors:

One point penalty for most simplification errors or arithmetic errors, after using the quotient rule correctly. An exception: 2 points penalty for making the type of erroneous cancelation shown at the bottom of page 15 of your textbook.

One point penalty for computing $\left(\frac{p}{q}\right)'$ as $\frac{pq'-p'q}{q^2}$ — that is, reversing the numerator terms, and thereby multiplying the result by -1.

Three point penalty for using the rule $(pq)' = p'q + pq'$ (which is correct but irrelevant).

Four point penalty (i.e., no credit) for more serious conceptual errors, i.e., if the student was clueless.

(4 points)

Green version: $p(x) = \left(x^3+3\right)^{17}$

White version: $q(x) = \left(x^2+x+1\right)^{12}$

Solutions for both versions: $\frac{d}{dx}\left[f(x)^r\right] = r\left[f(x)^{r-1}\right] f'(x)$, with

	green	white
$r =$	17	12
$f(x) =$	x^3+3	x^2+x+1
$f'(x) =$	$3x^2$	$2x+1$
correct answer:	$\boxed{51\left(x^3+3\right)^{16} x^2}$	$\boxed{12\left(x^2+x+1\right)^{11}(2x+1)}$
or:	$\boxed{51x^2\left(x^3+3\right)^{16}}$	$\boxed{12(2x+1)\left(x^2+x+1\right)^{11}}$
or:		$\boxed{(24x+12)\left(x^2+x+1\right)^{11}}$

1 pt penalty for $\quad 3\cdot 17x^2\left(x^3+3\right)^{16} \qquad 24x+12\left(x^2+x+1\right)^{11}$

2 pt penalty for

$$[rf(x)]^{r-1} f'(x) = \quad 3x^2\left(17x^3+51\right)^{16} \quad \left(12x^2+12x+1\right)^{11}(2x+1)$$

3 pt penalty for any of

	green	white
$r\left[f(x)\right]^{r-1} =$	$17\left(x^3+3\right)^{16}$	$12\left(x^2+x+1\right)^{11}$
$r\left[f'(x)\right]^{r-1} =$	$17\left(3x^2\right)^{16}$	$12\left(2x+1\right)^{11}$
$rf'(x)f(x) =$	$51x^2(x^3+3)$	$12(2x+1)(x^2+x+1)$