Quiz 3, 23 October 2009, 20 minutes, 1 page, 15 points. ANSWERS.

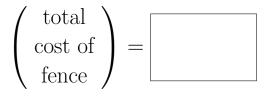
(6 points)

(o pomos)	
white version	green version
of $f(x)$, where $f(x) = x^4 - 8x^2$,	Find the absolute minimum value of $g(x)$, where $g(x) = \frac{1}{2}x^2 + x^{-1}$, on the interval $\frac{1}{2} \le x \le 2$. An- swer: $g(x) =$
Solutions	
4x(x-2)(x+2), which vanishes at	$g'(x) = x - x^{-2} = x^{-2}(x^3 - 1) = x^{-2}(x - 1)(x^2 + x + 1)$, which vanishes at $x = 1$. We need to check that point, and the end- points. Compute $g(1/2) = 2.125$, g(1) = 1.5, and $g(2) = 2.5$. The absolute minimum is 1.5]. Par- tial credit will be give for a value of 1, the value of x where the minimum occurs.

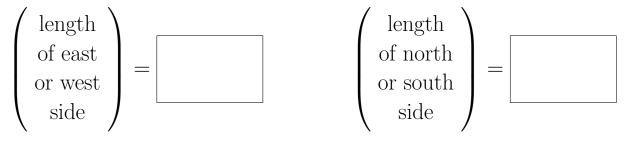
(9 points)

green version	white version
A rectangular garden, 120 square	A rectangular garden, 90 square
feet in area, is to be created by	feet in area, is to be created by
fencing in a portion of a field. For	fencing in a portion of a field. For
reasons that will not be explained	reasons that will not be explained
here, the north side of the garden	here, the different sides of the gar-
requires a different type of fenc-	den require different kinds of fenc-
ing material than the other three	ing material. The fence on the
sides. The fence on the north side	north side will cost three dollars
will cost two dollars per foot, and	per foot, the fence on the south
the fence on the other three sides	will cost two dollars per foot, the
of the rectangle will cost three dol-	fence on the east and west sides
lars per foot.	will cost one dollar per foot.

Find the smallest possible cost of the fence:



and the dimensions of the rectangle yielding that price.



Solutions. Let n, e, w, s be the cost, per foot, of fencing for the north, east, west, and south sides of the rectangle; and let N, E, W, S be the lengths of those fences. The the cost of the north fence is nN, and similarly for each of the other sides. Thus the total cost is C =

nN + eE + wW + sS.

green version	white version
n = 2	n = 3
e = 3	e = 1
w = 3	w = 1
s = 3	s = 2
C = 2N + 3E + 3W + 3S	C = 3N + E + W + 2S

However, it's a rectangle we're dealing with here, so N = S and E = W. Thus the total cost is C = (n + s)N + (e + w)E.

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That's the cost function, as a function of two variables; that equation is worth 1 point.

We're given the total area, a = NE. (Recognizing that was worth 1 point.)

$$NE = 120$$
 $NE = 90$

We can eliminate either one of the variables — either N or E — and then solve the problem in terms of the other variable.

If we eliminate E, we compute $E = aN^{-1}$, and so $C(N) = (n+s)N + (e+w)aN^{-1},$

where N is a variable and a, n, s, e, w are given constants. That's the cost function as a function of one variable; that's worth 2 points. Differentiating it correctly is worth another point:

$$C'(N) = (n+s) - (e+w)aN^{-2}.$$

Thus, in the two versions:

$E = 120N^{-1}$	$E = 90N^{-1}$
$C(N) = 5N + 720N^{-1}$	$C(N) = 5N + 180N^{-1}$
$C'(N) = 5 - 720N^{-2}$	$C'(N) = 5 - 180N^{-2}$

Finding the critical point(s) is worth another point: The only place where C'(N) is zero is where $n+s = (e+w)aN^{-2}$, or $N = \sqrt{\frac{(e+w)a}{n+s}}$.

$$N = 12 \qquad N = 6$$

Finishing the computation was worth the last 3 points: $E = aN^{-1}$, which yields $E = \sqrt{\frac{(n+s)a}{e+w}}$ and $C = 2\sqrt{(n+s)(e+s)a}$. $\boxed{\begin{array}{c} E = 10 \\ \hline C = 120 \end{array}} \quad \boxed{\begin{array}{c} E = 15 \\ \hline C = 60 \end{array}}$

If we eliminate N, we compute $N = aE^{-1}$, and so $C(E) = (n+s)aE^{-1} + (e+w)E$,

where E is a variable and a, n, s, e, w are given constants. That's the cost function as a function of one variable; it's worth 2 points. Differentiating it correctly is worth another point:

$$C'(E) = -(n+s)aE^{-2} + (e+w).$$

Thus, in the two versions:

$$N = 120E^{-1} \qquad N = 90E^{-1}$$

$$C(E) = 600E^{-1} + 6E \qquad C(E) = 450E^{-1} + 2E$$

$$C'(E) = -600E^{-2} + 6 \qquad C'(E) = -450E^{-2} + 2$$

Finding the critical point(s) is worth another point: The only place where C'(E) is zero is where $(n+s)aE^{-2} = e+w$, or $E = \sqrt{\frac{(n+s)a}{e+w}}$.

$$E = 10 \qquad E = 15$$

Finishing the computation was worth the last 3 points: $N = aE^{-1}$, which yields $N = \sqrt{\frac{(e+w)a}{n+s}}$, and $C = 2\sqrt{(n+s)(e+s)a}$. $\boxed{N=12}$ $\boxed{N=12}$ $\boxed{C=120}$ $\boxed{C=60}$