ANSWER KEY

Quiz 4, 17 November 2009, 20 minutes, 1 page, 15 points. Circle your answers.

(4 points)

White version: Find the derivative of $\ln \dfrac{x^3 e^{2x}}{(x+1)^{5/3}}$.

Pink version: Find the derivative of $\ln \dfrac{x^{3/7} e^{-x}}{(x-1)^5}$.

Solution.

white version	pink version
The given function can be rewritten as	
$3\ln x + 2x - \frac{5}{3}\ln(x+1)$	$\frac{3}{7}\ln x - x - 5\ln(x-1)$
and so its derivative is	
$\boxed{\dfrac{3}{x} + 2 - \dfrac{5}{3(x+1)}}$	$\boxed{\dfrac{3}{7x} - 1 - \dfrac{5}{x-1}}$

The correct answer consists of the sum of three terms. Students got 3 of the 4 points for giving 2 of the three terms correctly.

Some problems like this were worked in class. However, some students made this problem much harder for themselves, by not using the property $\ln(ab/c) = \ln a + \ln b - \ln c$. They simply applied $(\ln y)' = y'/y$, with $y = x^3 e^{2x}(x+1)^{5/3}$, computing y' with either the quotient rule or product rule.

(3 points)

White version: Find the derivative of $\dfrac{x^3 e^x}{(x+1)^{5/3}}$.

Pink version: Find the derivative of $\dfrac{x^{3/7} e^{-x}}{(x-1)^5}$.

Solution. Let y be the given function. Then the preceding problem was a computation of $(\ln y)' = \frac{y'}{y}$. Thus, all we need to do is multiply the result of the preceding problem $(\frac{y'}{y})$ times the function given in the current problem (y). This is similar to some problems that were worked in class, but some students made this much harder — e.g., they disregarded the similarity to the previous problem, and they used the quotient rule

$$\left(\frac{f}{g}\right)' \quad = \quad \frac{f'g - fg'}{g^2}$$

or some other such method to differentiate.

Actually, there was a misprint in the white version. The power of e accidentally got changed from e^{2x} to e^x. I will accept either version in the answer.

White version:

$$\boxed{\left(\frac{3}{x} + 2 - \frac{5}{3(x+1)}\right)\frac{x^3 e^{2x}}{(x+1)^{5/3}}} \text{ or}$$

$$\boxed{\left(\frac{3}{x} + 1 - \frac{5}{3(x+1)}\right)\frac{x^3 e^{x}}{(x+1)^{5/3}}}$$

Using the quotient rule:

$$f = x^3 e^x, \qquad g = (x+1)^{5/3}$$

$$f' = (x^3 + 3x^2)e^x, \qquad g' = \frac{5}{3}(x+1)^{2/3}$$

$$\frac{f'g - fg'}{g^2} \quad = \quad \frac{(x^3+3x^2)e^x(x+1)^{5/3} - \frac{5}{3}x^3 e^x (x+1)^{2/3}}{(x+1)^{10/3}}$$

$$= \frac{\left[(x^3+3x^2)(x+1) - \frac{5}{3}x^3\right] e^x}{(x+1)^{8/3}} = \boxed{\frac{\left(x^4+\frac{7}{3}x^3\right) e^x}{(x+1)^{8/3}}}.$$

Pink version: $\boxed{\left(\frac{3}{7x} - 1 - \frac{5}{x-1}\right)\frac{x^{3/7}e^{-x}}{(x-1)^5}}$

Using the quotient rule:

$$f(x) = x^{3/7}e^{-x}, \qquad g(x) = (x-1)^5$$

$$f'(x) = \left(\frac{3}{7}x^{-4/7} - x^{3/7}\right)e^{-x}, \qquad g'(x) = 5(x-1)^4$$

$$\frac{f'g-fg'}{g^2} = \frac{\left(\frac{3}{7}x^{-4/7} - x^{3/7}\right)(x-1)^5e^{-x} - 5x^{3/7}(x-1)^4e^{-x}}{(x-1)^{10}}$$

$$= \frac{\left[\left(\frac{3}{7}-x\right)(x-1) - 5x\right]x^{-4/7}(x-1)^4e^{-x}}{(x-1)^{10}} = \boxed{\frac{-7x^2-25x-3}{7(x-1)^6e^xx^{4/7}}}$$

(4 points)

white version	pink version
$\int(7+x^{-1}+2x^{-2})dx =$	$\int(-7+2x^{-1}+x^{-3})dx =$
Solution	
$\boxed{7x+\ln\lvert x\rvert - 2x^{-1}+C}$	$\boxed{-7x+2\ln\lvert x\rvert - \frac{1}{2}x^{-2}+C}$

In both cases, I will deduct 1 point for omitting the "+C", but no penalty for omitting the absolute values.

(4 points)

white version	pink version
$\int x^3(x^2+1)^7 dx =$	$\int x^5(x^3-1)^4 dx =$

Solution. We'll use these substitutions:

$u = x^2+1$ $du = 2x dx$ $\frac{1}{2}du = x dx$ $x^2 = u-1$	$u = x^3-1$ $du = 3x^2 dx$ $\frac{1}{3}du = x^2 dx.$ $x^3 = u+1$

One point for getting that far — i.e., for recognizing the need for a substitution, and identifying it correctly. Now here is the main computation:

$\int x^3(x^2+1)^7 dx$ $= \int x^2(x^2+1)^7(x dx)$ $= \int (u-1)u^7 \cdot \frac{1}{2}du$	$\int x^5(x^3-1)^4 dx$ $= \int x^3(x^3-1)^4(x^2 dx)$ $= \int (u+1)u^4 \cdot \frac{1}{3}du$

(One point for that step, i.e., for correctly substituting into the integral)

$= \frac{1}{2}\int (u^8 - u^7)du$ $= \frac{u^9}{18} - \frac{u^8}{16} + C$	$= \frac{1}{3}\int (u^5 + u^4)du$ $= \frac{u^6}{18} + \frac{u^5}{15} + C$

(One point for that step, i.e., for finding the antiderivative in the substituted variable)

$= \boxed{\frac{(x^2+1)^9}{18} - \frac{(x^2+1)^8}{16} + C}$	$= \boxed{\frac{(x^3-1)^6}{18} + \frac{(x^3-1)^5}{15} + C}$