Quiz 1 ANSWER KEY
7 September 2009, 20 minutes, 1 page, 15 points.

General notes on partial credit: A wrong answer will receive partial credit if some work is shown that demonstrates the student had some idea of what he or she was doing; how much credit will depend on how much sense the grader can make of the student's work. Zero credit will be given if the answer is wrong and no work is shown, or the work that is shown is incomprehensible. Scores will be integers only — i.e., no fractional points. In general, an arithmetic error (such as 2 times 3 equals 5) cost 1 point; a conceptual error (such as $x^2 \cdot x^3$ equals x^6) could cost more. A transcription error — i.e., copying an expression incorrectly from one line of your computation to the next line — cost 1 point.

White version:

(4 points) Simplify $\dfrac{(2x^2)(3x^3)^2}{\sqrt{x^8}} =$ $\boxed{18x^4}$

Solution steps (optional): $2 \cdot 3 \cdot 3 \cdot x^{2+(3\cdot 2)-(8/2)}$

Yellow version:

(4 points) Simplify $\dfrac{(2x^3)(3x^2)^3}{\sqrt{x^6}} =$ $\boxed{54x^6}$

Intermediate solution steps (optional): $2 \cdot 3 \cdot 3 \cdot 3 \cdot x^{3+(2\cdot 3)-(6/2)}$

Some notes about partial credit:

You should know that $(2x^2)(3x^3)^2$ means $(2x^2)\left[(3x^3)^2\right]$, and not $\left[(2x^2)(3x^3)\right]^2$; interpreting it as the latter cost 2 points. Similarly, you should know that $(2x^3)(3x^2)^3$ means $(2x^3)\left[(3x^2)^3\right]$, and not $\left[(2x^3)(3x^2)\right]^3$; interpreting it as the latter cost 2 points.

$18\sqrt{x^8}$ should be simplified to $18x^4$; omitting to do this cost 1 point. Leaving $\dfrac{18x^8}{\sqrt{x^8}}$ unsimplified cost 2 points.

$(3x^3)^2$ is equal to $9x^6$; computing it instead as $3x^6$ or as $9x^5$ cost 1 point. Similarly, $(3x^2)^3$ is equal to $27x^6$; computing it instead as $3x^6$ or as $27x^5$ cost 1 point.

An expression such as $\dfrac{18x^2 \cdot x^6}{x^4}$ cost 2 points, because there are two further simplifications needed.

White version:

(4 points) Solve the inequality $x^2 - 5x + 6 < 0$.

$2 < x < 3$
or
$(2, 3)$

Solution steps (optional): $(x-2)(x-3) < 0$, so $x - 3 < 0 < x - 2$.

Yellow version:

(4 points) Solve the inequality $x^2 + 5x + 6 < 0$.

$-3 < x < -2$
or
$(-3, -2)$

Intermediate solution steps (optional): $(x+2)(x+3) < 0$, so $x + 2 < 0 < x + 3$.

Common errors:

Using brackets instead of parentheses — e.g., writing $[2, 3]$ instead of $(2, 3)$ — cost 1 point.

Writing $(-2, -3)$ instead of $(-3, -2)$ cost 1 point. Note that $-3 < -2$, so $(-2, -3)$ does not exist.

The product of two numbers is negative only if one of those two factors is negative and the other is positive. But several students seemed to be reasoning that the product of two numbers is negative if both of those numbers are negative. Thus they arrived at the statement

$$x < 2 \qquad \text{and} \qquad x < 3 \qquad\qquad (a)$$

or the statement

$$x < -2 \qquad \text{and} \qquad x < -3 \qquad\qquad (b)$$

as an intermediate step. For that conceptual error I deducted 2 points; I deducted further points if the consequences of that intermediate step were not worked out correctly (and usually they weren't). Statement (a) is actually equivalent to "$x < 2$" (since that statement *implies* $x < 3$), and statement (b) is actually equivalent to "$x < -3$" (since that statement *implies* $x < -2$). Thus, for the full 2 points, a student would have to answer $x < 2$ or $(-\infty, 2)$ (white paper version) or $x < -3$ or $(-\infty, -3)$ (yellow paper version).

Coming up with the correct two roots (2 and 3, or -2 and -3), but doing the wrong thing with them, was worth 1 point.

Coming up with the wrong roots altogether was generally worth 0 points, unless the source of the error was something simple that was easily discernible by the grader (e.g., a transposition error in the quadratic formula).

White version:

(3 points) The distance between $(-1, 5)$ and $(3, 2)$ is $\boxed{5}$

Solution steps (optional): $\sqrt{(-1-3)^2+(5-2)^2}$

Yellow version:

(3 points) The distance between $(-1, 4)$ and $(3, 7)$ is $\boxed{5}$

Intermediate solution steps (optional): $\sqrt{(-1-3)^2+(4-7)^2} = \sqrt{16+9}$.

Common errors:

An answer of 25 (i.e., forgetting to take square roots) was worth 2 points.
The correct formula for the distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}.$$

One point was given for using any of these incorrect distance formulas:

$$\sqrt{(x_1-x_2)+(y_1-y_2)}$$
$$\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$$
$$\sqrt{(x_1+x_2)^2-(y_1+y_2)^2}$$
$$\sqrt{(x_1-y_2)^2+(y_1-x_2)^2}$$
$$\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$$

White version:

(4 points) Give an equation for the line through $(-1, 5)$ and $(3, 2)$:

$y - 5 = -\frac{3}{4}(x+1)$ **or**
$y - 2 = -\frac{3}{4}(x-3)$ **or**
$y = -\frac{3}{4}x + \frac{17}{4}$ **or**
$3x + 4y = 17$

Solution steps (optional): $m = \dfrac{5-2}{-1-3} = \dfrac{-3}{4}$

Yellow version:

(4 points) Give an equation for the line through $(-1,4)$ and $(3,7)$:

$$y-4=\tfrac{3}{4}(x+1) \quad \textbf{or}$$
$$y-7=\tfrac{3}{4}(x-3) \quad \textbf{or}$$
$$y=\tfrac{3}{4}x+\tfrac{19}{4} \quad \textbf{or}$$
$$-3x+4y=19$$

Intermediate solution steps (optional): $m=\dfrac{4-7}{-1-3}=\dfrac{-3}{-4}=\dfrac{3}{4}$

Partial credit:

One point if the student gave the equation for some line, but not a line with any apparent connection to the given data.

Two points if the student gave the equation for a line with the right slope, but not passing through either of the given points.

Two points if the student gave the equation for a line passing through one of the given points but not the other one.