2.6.75.c 4. [8] Determine all values of a for which g is continuous. Justify your answer.

(a) $g(x) = \begin{cases} x^2 + x & \text{if } x < 1 \\ a & \text{if } x = 1 \\ 3x + 5 & \text{if } x > 1 \end{cases}$

$\lim_{x \to 1^-} g(x) = 2$ and $\lim_{x \to 1+} g(x) = 8$.

Those are different, no matter what we choose for a.

none, i.e. no such values of a

3.4.65 (b) $g(x) = \begin{cases} \dfrac{1 - \cos x}{2x} & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$

$\lim_{x \to 0} g(x) = 0$, so we need $a = 0$

3.2.58 5. [6] Determine the value of $\lim_{h \to 0} \dfrac{(1+h)^8 + (1+h)^3 - 2}{h}$ by finding $f'(a)$ for an appropriate f and a.

Let $f(x) = x^8 + x^3$ and $a = 1$

Then $f'(x) = 8x^7 + 3x^2$, and the desired limit is
$f'(1) = 8 + 3 = 11$.

(Problem can also be done with $f(x) = (x+1)^8 + (x+1)^3$ and $a = 0$; this also yields 11.)

3.4.13 6. [3] Find $\lim_{x \to 2} \dfrac{\sin(x-2)}{x^2 - 4}$

$$= \left(\lim_{x \to 2} \frac{\sin(x-2)}{x-2}\right)\left(\lim_{x \to 2} \frac{1}{x+2}\right)$$

$$= \left(\lim_{u \to 0} \frac{\sin u}{u}\right)\left(\lim_{x \to 2} \frac{1}{x+2}\right) = 1 \cdot \frac{1}{4} = \frac{1}{4}$$

by factoring $x^2 - 4 = (x-2)(x+2)$
and substituting $u = x - 2$.