Name (please print): **ANSWER KEY**
Math 150B Quiz 7, Thursday 26 March 2009, 1 page, 15 points, 15 minutes

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(5 points) $\int \sin^3 x\ \cos^2 x\ dx =$

Solution. Substitute $u = \cos x$. Then $du = -\sin x\, dx$, so the given integral is

$$= -\int (1-\cos^2 x)(\cos^2 x)(-\sin x)dx \ = \ -\int (1-u^2)u^2\, du$$

$$= \int (u^4 - u^2)du \ = \ \frac{u^5}{5} - \frac{u^3}{3} + C \ = \ \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}.$$

The most common error was a sign error, for which I deducted one point.

(5 points) $\int \sin 3x\ \cos 2x\ dx =$

Solution.

$$= \int \frac{\sin(3x+2x)+\sin(3x-2x)}{2}dx \ = \ \frac{1}{2}\int [\sin 5x + \sin x]\, dx$$

$$= \frac{1}{2}\left[-\frac{1}{5}\cos 5x - \cos x\right] + C \ = \ \boxed{-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C}.$$

Special note about partial credit on this problem: When I covered this topic in class, I had not noticed that it had been omitted from the syllabus. But many students apparently studied only from the syllabus, not from what was covered in class. Since I never said explicitly whether to study what was covered in class but omitted from the syllabus, it was not really clear what students should do, and there really is no completely fair way to deal with it — should I reward the students who did study it? Or penalize the students who did not? I'll try to avoid this kind of dilemma in the future. But in any case I have decided that the closest approximation to fairness that I could find was to grade this problem very leniently. So here is my grading scale: 5 points for a correct answer; 4 points for a minor error such as a sign error; and 3 points for any other answer at all, including no answer.

(5 points) $\int x^2 \ln x \, dx =$

Solution. The step marked (IP) is integration by parts. The given expression is

$$= \int \left(\frac{x^3}{3}\right)' (\ln x)\, dx \overset{(IP)}{=} \left(\frac{x^3}{3}\right)(\ln x) - \int \left(\frac{x^3}{3}\right)(\ln x)'\, dx$$

$$= \left(\frac{x^3}{3}\right)(\ln x) - \int \left(\frac{x^3}{3}\right) \cdot \frac{1}{x} dx = \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx = \boxed{\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C}$$

If you want to use the formula $\int u'v\, dx = uv - \int uv'\, dx$, in this problem you should use $u = x^3/3$ and $v = \ln x$. Some students used $u = x^2$ and $v = \ln x$, which doesn't really work here.
