Name (please print): **ANSWER KEY**
Math 150B Quiz 6, Tuesday 17 March 2009, 1 page, 15 points, 15 minutes

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(5 points) $\displaystyle\int \frac{1+x}{1+x^2}dx =$

Solution. $\displaystyle = \int \frac{1}{1+x^2}dx + \frac{1}{2}\int \frac{xdx}{1+x^2} = \arctan x + \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2}$
$= \boxed{\arctan x + \frac{1}{2}\ln(1+x^2) + C}$.

One of the most common errors was to change the $\frac{1}{2}$ ti a 2 in the answer; I charged one point for that.

I gave 3 points for any answer that involved $\arctan x + C$, or for any answer that involved $\ln(1+x^2) + C$.

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points5 $\displaystyle\int \frac{dx}{\sqrt{4-x^2}} =$

Solution. Substitute $2u = x$. Then $2du = dx$, so we get

$$2\int \frac{du}{\sqrt{4-4u^2}} = 2\int \frac{du}{2\sqrt{1-u^2}} = \arcsin u + C = \boxed{\arcsin \frac{x}{2} + C}.$$

I gave 3 points for any answer of the form $p \arcsin qx + C$ for some constants p and q, or 2 points for any answer that at least involved arcsin in some fashion.

(5 points) $\displaystyle\int_0^{\ln 2} \frac{\sinh x}{2+\cosh x}dx =$

Solution. Substitute $u = 2 + \cosh x$; then $du = \sinh x\, dx$. We also compute

x	e^x	$\cosh x$	$u = 2 + \cosh x$
$\ln 2$	2	$\frac{2+\frac{1}{2}}{2} = \frac{5}{4}$	$\frac{13}{4}$
0	1	1	3

Hence

$$\int_0^{\ln 2} \frac{\sinh x}{2+\cosh x} dx = \int_3^{13/4} \frac{du}{u} = [\ln|u|]_3^{13/4} = \ln\frac{13}{4} - \ln 3 = \boxed{\ln\left(\frac{13}{12}\right)}.$$

Another way to compute this was

$$\int \frac{\sinh x}{2+\cosh x} dx = \int \frac{du}{u} = \ln|u| + C$$

and so we arrive at $[\ln(2+\cosh x)]_0^{\ln 2}$; that expression was worth 3 points. It can be expanded to

$$\ln(2+\cosh\ln 2) - \ln(2+\cosh 0)$$

which is not simplified enough. Generally I took off one or two points for insufficient simplifying. One of the most common errors was to omit one or both of the "2+" from that last line; that leads to one of the incorrect answers $\ln(5/12)$ or $\ln(13/4)$ or $\ln(5/4)$, for any of which I gave 3 points.