Name (please print): **ANSWER KEY**
Math 150B Quiz 5, Tuesday 24 February 2009, 1 page, 15 points, 15 minutes

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(4 points) $\dfrac{d}{dx}\left(x^x\right) =$

Solution. (This exact same problem was discussed in class.) Let $y = x^x$. Then $\ln y = x \ln x$. Differentiating both sides yields $\dfrac{y'}{y} = 1 + \ln x$. Then $y' = \boxed{x^x(1+\ln x)}$. I gave 3 points for an answer of $e^{x \ln x}(1 + \ln x)$ (which is equal to the correct answer but is not simplified enough). I gave 1 point for $x^x \ln x$, which is sort of on the right track.

(4 points) $\displaystyle\int \frac{e^x}{e^x+3}\, dx =$

Solution. Let $u = e^x + 3$; then $du = e^x dx$, so the integral is equal to $\displaystyle\int \frac{du}{u}$, for which I gave 1 pont. That evaluates to $\ln|u| + C$, for which I gave 2 points. Now resubstitutiong the original variable yields $\ln|e^x+3|+C$, for which I gave 3 points. Finally, since e^x is always positive, so is $e^x + 3$, and therefore the absolute value signs should be omitted. Thus, for full credit, the answer is $\boxed{\ln(e^x+3)+C}$.

One error that some students made was to rewrite $\dfrac{e^x}{e^x+3}$ as $\dfrac{e^x}{e^x} + \dfrac{e^x}{3}$, but of course that's very very wrong.

(3 points) $\displaystyle\int \frac{e^x+3}{e^x}\, dx =$

Solution. $= \int \left(1 + 3e^{-x}\right) dx = \boxed{x - 3e^{-x} + C}$. I deducted 1 point for omitting the $+C$ or for making a sign error in any of several places. Most other incorrect answers displayed a severe lack of understanding, and merited no partial credit at all. Among those no-credit answers, the most common one was $4x + C$, an answer that students would arrive at if they decided that $\displaystyle\int \frac{1}{e^x} dx$ was equal to $\ln(e^x) + C$. It's not, and this is an instance of a common error that I warned about in class. It is true that $\displaystyle\int \frac{du}{u} = \ln|u| + C$, but $\displaystyle\int \frac{dx}{u}$ generally is *not* equal to $\ln|u| + C$.

(4 points) $\displaystyle \frac{d}{dx} \frac{\sqrt[3]{x+1}\,(x^3-1)^5}{2^x} =$

Solution. Let the given fraction be y; then

$$\ln y \quad = \quad \frac{1}{3}\ln(x+1) + 5\ln(x^3-1) - x\ln 2.$$

Some students gave the right side of that equation as their answer; I gave 1 point for it. Now Differentiate both sides; thus

$$\frac{y'}{y} \;=\; (\ln y)' \quad = \quad \frac{1}{3(x+1)} + \frac{5(3x^2)}{x^3-1} - \ln 2.$$

Again, some students gave the right side of that as their answer; I gave 3 points for that, or 2 points for a close approximation to it. Now multiply both sides through by y; thus

$$y' \;=\; \boxed{\left[\frac{1}{3(x+1)} + \frac{15x^2}{x^3-1} - \ln 2\right] \frac{\sqrt[3]{x+1}\,(x^3-1)^5}{2^x}}.$$

This problem was fairly easy if the method above was used, and much harder and more prone to computational errors if one instead used the quotient rule, which many students did — i.e.,

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2} \qquad \text{where}$$

$$\begin{array}{rclcrcl} f & = & \sqrt[3]{x+1}\,(x^3-1)^5 & & g & = & 2^x \\ f' & = & \frac{1}{3}(x+1)^{2/3}(x^3-1)^5 + 15x^2\sqrt[3]{x+1}(x^3-1)^4 & & g' & = & 2^x \ln x. \end{array}$$

The answer can be expressed as

$$\boxed{\frac{\frac{1}{3}(x+1)^{-2/3}(x^3-1)^5 + 15x^2(x^3-1)^4\sqrt[3]{x+1} - (\ln 2)(x^3-1)^5\sqrt[3]{x+1}}{2^x}}.$$

Note that

$$\frac{2^x}{(2^x)^2} = \frac{1}{2^x},$$

so the left side of that equation is not simplified enough.