**Problem 1.** Find an orthonormal basis of $\mathbb{R}^3$ containing $(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}), (\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 0)$.

**Solution.** The two vectors are orthogonal and have length 1. The third vector in the orthonormal basis can be found as the cross-product of these two vectors.

**Problem 2.** Find a basis of the column space of the following matrix:

$$
\begin{bmatrix}
2 & 4 & 3 & 6 & 8 \\
1 & 2 & 3 & 4 & 5 \\
3 & 7 & 6 & 9 & 12 \\
\end{bmatrix}
$$

**Solution.** Reduce the matrix to the RREF, find which columns contain pivots. The corresponding columns of the original matrix form a basis of the column space.

**Problem 3.** (a) Suppose that the system of equations $Ax = \vec{0}$ has 3 pivotal variables, 10 independent variables. What are the rank and the nullity of the matrix $A$?

**Solution.** rank is 3, nullity is 10.

(b) Let $A = 
\begin{pmatrix}
1 & 4 & 0 & 0 & 8 \\
0 & 0 & 1 & 0 & 5 \\
0 & 0 & 0 & 1 & 12 \\
\end{pmatrix}$. Find a basis of the null-space of $A$.

**Solution.** The matrix is in the row echelon form already. The pivotal variables are $x_1, x_3, x_4$, independent variables are $x_2, x_5$. The vector form of the solution set is

$$
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{pmatrix} =
\begin{pmatrix}
-4 \\
1 \\
0 \\
0 \\
0 \\
\end{pmatrix}
+ 
\begin{pmatrix}
x_2 \\
x_5 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\begin{pmatrix}
-8 \\
0 \\
-5 \\
-12 \\
1 \\
\end{pmatrix}
$$

So a basis of the solution set consists of

$$
\begin{pmatrix}
-4 \\
1 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
-8 \\
0 \\
-5 \\
-12 \\
1 \\
\end{pmatrix}
$$