Combinatorial and Geometric Group Theory

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Contents

Actions of pointed Hopf algebras on quantum torus

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Let O_q the associative algebra with a unit element over a field k generated by elements $X_1^{\pm 1}, \ldots, X_r^{\pm 1}, X_{r+1}, \ldots, X_n$ subject to defining relations $X_i X_i^{-1} = X_i^{-1} X_i = 1, \quad 1 \leqslant i \leqslant r$ and $X_i X_j = q_{ij} X_j X_i, \quad 1 \leqslant i, j \leqslant n$. Here q_{ij} are element of k such that $q_{ii} = q_{ij} q_{ji} = 1$ for all i, j. The algebra O_q is an algebra of *quantum polynomials*. It is a *generic* algebra of quantum polynomials if all multiparameters q_{ij} with $1 \leqslant i < j \leqslant n$, are independent in the multiplicative abelian group k^* of the field k. The algebra O_q can be considered as a coordinate algebra of product of a quantum torus and a quantum plane [BrG, M].

In non-commutative algebraic geometry an action of a "finite quantum group" on a quantum space means an action of some finite dimensional Hopf algebra H on O_q. In my talk I shall consider the case when H is a pointed Hopf algebra. It is shown that there exists a class of standard cocommutative pointed finite dimensional Hopf algebra acting on O_q. An action of H is a composition of Hopf algebra homomorphism from H onto some standard algebra and an action of the standard one on O_q. It follows that an action of H on O_q is reduced to action of some automorphism group and some skew derivations of O_q. Moreover the subalgebra of invariants of this action is left and right Noetherian and O_q is finitely generated left and right module over the subalgebra of invariants.

In the case when the number n of variables is at least three was considered in [A]. It is interesting to mention that in the case $n = r = 2$ a classification of automorphism group of O_q is similar to a classification of two-dimensional crystallographic groups.

[A] Artamonov V. A., Pointed Hopf algebras acting on quantum polynomials, J. Algebra 259(2003), N 2, 323-352.

[BrG] Brown K. A., Goodearl K. R., Lecture on algebraic quantum groups. Birkhäuser, Basel, Boston, 2002.

[M] Montgomery,S.: Hopf Algebras and Their Actions on Rings, Regional Conf. Ser. Math. Amer. Math. Soc., Providence RI, 1993.

Relatively hyperbolic groups are C*-simple

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We characterize relatively hyperbolic groups whose reduced C^*-algebra is simple as those, which have no non-trivial finite normal subgroups.

On embeddings of free Burnside groups of odd exponent $n \geq 1003$

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We proved the following theorem: for each odd number $n \geq 1003$ each non cyclic subgroup of the 2-generated free n-Burnside group $B(2, n)$ contains a subgroup isomorphic to the free n-Burnside group $B(\infty, n)$ of countable rank. This theorem, which strengthens the earlier result obtained by the author for $n > 10^{80}$, extends the class of those free Burnside groups for which the hypothesis of A. Yu. Olshanskii formulated in Kourovka Notebook (8.53 b) has a positive answer.

Gradings by Groups on Classical Simple Algebras

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If A is an algebra over a field F and G a group then a very general fact is that the gradings of A by G are equivalent to the natural actions of a Hopf algebra H=(FG)* dual to the group algebra FG. The structure of the coproduct in H is not simple but in certain cases H is just the group algebra of the group of characters of G, in some others H is a restricted enveloping algebra of a Lie algebra of derivations of FG with values in F, etc. In our talk we would like to discuss the methods arising and the results on the gradings of simple algebras of various classes which have been obtained using these methods.

Global fixed points for actions on CAT(0) spaces

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A group G has Farb's property FA_n if every action of G on every CAT(0) n-complex has a global fixed point. This is a generalization of Serre's property FA with applications to representation theory and to the study of complex of groups decompositions. We will consider many examples.

Residually torsion-free-nilpotent groups

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I will talk about various aspects of residually torsion-free-nilpotent groups, in particular various algorithmic problems.

Thick metric spaces, relative hyperbolicity, and quasi-isometric rigidity.

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In this talk we will introduce a new quasi-isometry invariant of metric spaces which we call thick. We show that any thick metric space is not (strongly) relatively hyperbolic with respect to any non-trivial collection of subsets. Further, we show that the property of being (strongly) relatively hyperbolic with thick peripheral subgroups is a quasi-isometry invariant. The class of thick groups includes many important examples such as mapping class groups of all surfaces (except those few that are virtually free), the outer automorphism group of the free group on at least 3 generators, $SL(n, Z)$ with $n > 2$, and others. We shall also discuss some ways in which thick groups behave rigidly under quasi-isometries.

Rips construction and Kazhdan property (T)

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We note that the small cancellation theory over hyperbolic groups yields an attractive version of the Rips construction: for each non-elementary hyperbolic group H and a finitely presented group Q there is a short exact sequence

$$1 \to N \to G \to Q \to 1$$

where N is a quotient of H, and G is hyperbolic. This also has a relatively hyperbolic version where H, G are relatively hyperbolic and Q is finitely generated. Sample applications:

1) Any finitely generated group embeds as a finite index subgroup in $Out(N)$ where N has property (T) (or more generally a quotient of any given non-elementary relatively hyperbolic group). One can also arrange that $Out(Aut(N)) = 1$.

2) There exists a large torsion-free hyperbolic group G and an element g in G such that the group $\langle G \,|\, g^n \rangle$ is not large for all odd n.

3) There is a torsion-free hyperbolic group that is representation rigid, but not representation superrigid.

4) Property (T) is not recursively recognizable in the class of hyperbolic groups.

A universal, minimal, non-solvable subgroup of PLo(I) and of R. Thompson's Group F

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There has been research into the question of whether the group $PL_o(I)$ (piecewise-linear, orientation preserving homeomorphisms of the unit interval under the operation of composition) admits a universal minimal non-solvable subgroup. We will outline an argument which demonstrates that the answer to this is "Yes." We give a description of this group W, and we note the following two corollaries; first, R. Thompson's group F also contains a copy of W as a universal minimal non-solvable subgroup, and second, given any non-solvable subgroup N of $PL_o(I)$ or F, and any solvable subgroup S of $PL_o(I)$ or F, we have that S embeds in N.

Periodic points of self-maps of algebraic varieties and their applications to group theory.

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Together with Mark Sapir, we proved that all mapping tori of free group automorphisms are residually finite. The proof uses periodic points of self-maps of algebraic varieties over finite fields. I will explain the main ideas of the proof without going into the technical algebraic geometry details.

A generalization of the prime geodesic theorem to counting conjugacy classes of free subgroups.

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The prime geodesic theorem gives an asymptotic formula (as x tends to infinity) for the number of conjugacy classes of elements of $pi_1(M)$ of geometric translation length at most x where M is a hyperbolic manifold. We generalize this formula in the direction of counting conjugacy classes of finitely generated free subgroups of $pi_1(M)$ with general geometric constraints. There are many open questions.

Hurewicz Theorem for Nagata-Assouad dimension

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Given a function $f\colon X \to Y$ of metric spaces, its *asymptotic dimension* $asdim(f)$ is the supremum of $asdim(A)$ such that $A \subset Y$ and $asdim(f(A)) = 0$. Our main result is

Theorem A: $asdim(X) \leq asdim(f) + asdim(Y)$ for any coarse function $f\colon X \to Y$.

As an application we prove

Theorem B: $asdim(G) \leq asdim(K)+asdim(H)$ for any short exact sequence $1 \to K \to G \to H \to 1$ of countable groups.

Both Theorems A and B generalize results of Bell and Dranishnikov in which f is Lipschitz and X is geodesic and G, K are finitely generated, respectively. We provide analogs of A and B for linearly controlled asymptotic dimension and Nagata-Assouad dimension.

The Dimension of the Torelli Group for $Out(F_n)$

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Abelianizing induces a natural homomorphism from $Out(F_n)$ to $GL_n(Z)$. This homomorphism is surjective. In analogy to the situation in mapping class groups, we call its kernel the Torelli subgroup of $Out(F_n)$, and we denote it by T_n. We show:

Theorem. For $n \geq 3$, the following hold:

1. The Torelli subgroup T_n has an Eilenberg-Mac Lane complex of dimension $2n-4$.

2. Its integral homology in top dimension, $H_{2n-4}(T_n; Z)$, is not finitely generated. In particular, T_n is not of type FP_{2n-4}.

Our approach is purely geometric: we construct an Eilenberg-Mac Lane space of dimension $2n-3$ as a quotient of the spine of Outer Space. Then, we use combinatorial Morse theory to show that it is homotopy equivalent to a space of dimension $2n-4$. Finally, we exhibit explicitly an infinite family of independent homology classes.

We note that our methods also yield a geometric proof of the classical fact (due to Magnus) that T_n is finitely generated.

We also note that a spectral sequence argument allows one to deduce similar statements for the Torelli subgroup of $Aut(F_n)$.

Toward Outer Space for Right Angled Artin Groups

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Right-angled Artin groups are finitely generated groups whose only relations are commutator relations between pairs of generators. This class of groups may be thought of as interpolating between free groups (no generators commute) and free abelian groups (all generators commute). Thus, automorphism groups of right-angled Artin groups interpolate between $Aut(F_n)$ and $GL(n, Z)$. We study the automorphism group of a right-angled Artin group A in the case where the defining graph is connected and triangle-free. We establish some algebraic properties of $Aut(A)$ and construct a candidate "outer space" by considering actions of A on 2-dimensional CAT(0) complexes.

Strongly bounded groups

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A group is strongly bounded if it has no isometric action on any metric space with unbounded orbits. The notion has only emerged since a recent paper by G. Bergman, although there are plenty of examples, including infinite symmetric groups and infinite powers of finite simple groups, endowed with the discrete topology.

Boundary of foliations

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I shall talk on foliations of Hadamard manifolds with a short 2nd fundamental form. Leaves of such foliations are Hadamard manifolds and we can embed their ideal boundaries into the ideal boundary of the manifold. Some examples and geometric properties will be provided.

Second order Dehn functions of Pride Groups

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A class of groups (known as Pride groups) given by presentations in which each defining relator involves at most two types of generators was introduced in [1]. I will talk about the generators of the second homotopy module of such groups and from this obtain an upper bound for their second order Dehn function.

[1] Stephen J. Pride. The (co)homology of groups given by presentations in which each defining relator involves at most two types of generators. J. Austral. Math. Soc. Ser. A, 52(2), 205-218, 1992

Free actions of elementary abelian groups on tori.

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The main result we prove is the following:
Theorem: Suppose that $\mathbb{Z}_p^h$ (p prime) acts freely on an n-dimensional torus, then h is less than or equal to n.
This generalizes a result due to Goncalves D.L. and Vieira J.P. who proved the above result in case h is smaller than 4.
The proof boils down to finding an upper bound on the minimal number of generators of Bieberbach group, with an elementary abelian holonomy group. It turns out to be essential to make a difference between the cases $p = 2$ and $p > 2$.

Multiplicative groups of Engel associative algebras

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Let R be an associative ring. Let $[R]$ denote the associated Lie ring of R (with $[a, b] = ab - ba$) and let $U(R)$ denote the multiplicative group of R. It is known that if $[R]$ is nilpotent of class c then the group $U(R)$ is nilpotent of class at most c (Gupta and Levin, 1983) and if $[R]$ is metabelian then $U(R)$ is metabelian (Krasilnikov, 1992, and independently Sharma and Srivastava, 1992). Our main result is as follows.
Theorem. *Over each field of characteristic* 2 *there exists an associative algebra* R *such that its associated Lie algebra* $[R]$ *is* 3*-Engel but its multiplicative group* $U(R)$ *is not* 3*-Engel.*

Complexity and decidability results for some free inverse monoids

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Margolis and Meakin have shown that the word problem for free inverse monoids modulo a finite idempotent presentation is decidable. Later another, more direct proof was given by da Silva. In the lecture we present a proof for this result using rewriting techniques over finite subsets of trees which leads to optimal algorithms for solving the word problem in the uniform and non-uniform setting.

Moreover, we show that the membership in rational subsets is decidable for these monoids. This implies that the generalized word problem is decidable, too. As matter of fact, our techniques can be extended to cope with virtually free groups as starting point. (This is interesting, for example because virtually free groups correspond exactly to the class of groups where the monadic second order logic of the Cayley graphs is decidable).

The results are closely related to the journal version for the MFCS 2005 contribution of Lohrey and Ondrusch.

Embedding of groups into the product of binary trees

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Every hyperbolic group of asymptotic dimension n can be embedded quasi-isometrically into the product of n binary trees.

Golod-Shafarevich groups with property (T) and Kac-Moody groups

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A finitely generated group is called a Golod-Shafarevich group if it has a presentation $< X|R >$ with the following property:

There exists a prime number p and a real number $0 < t_0 < 1$ such that $1 - |X|t_0 + \sum_{i=1}^{\infty} r_i t_0^i < 0$ where r_i is the number of defining relators which have degree i with respect to the dimension p-series.

Golod-Shararevich groups are always infinite and moreover behave like free groups in many ways. On the other hand, it is not clear if a Golod-Shafarevich

group must have 'a lot of' finite quotients. The following is a well-known question of this type:

Is it true that Golod-Shafarevich groups never have property (tau)?

By a recent work of Lackenby, an affirmative answer to this question would have implied Thurston's virtual positive Betti number conjecture for arithmetic hyperbolic 3-manifolds. In this talk I will show that the answer to the above question is negative in general. Explicit examples of Golod-Shafarevich groups with property (tau) (in fact, (T)) are given by lattices in certain Kac-Moody groups over finite fields.

The Action of Thompson's Group on a CAT(0) Boundary

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One way to prove that Thompson's group is non-amenable is to show that it acts isometrically on a proper CAT(0) space without fixing any points at infinity.

I will consider a natural CAT(0) cubical complex on which F acts and show that it fixes an arc at infinity of Tits length $\pi/2$.

The talk will also include a description of the CAT(0) boundary for F in terms of semigroup pictures.

Twisted Burnside-Frobenius theorem for discrete groups

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It is proved for a wide class of groups including polycyclic and finitely generated polynomial growth groups that the Reidemeister number of an automorphism is equal to the number of finite-dimensional fixed points of induced map on the unitary dual space, if one of these numbers is finite. This theorem is a natural generalization to infinite discrete groups of the classical Burnside-Frobenius theorem. On other hand it has important consequences in dynamics and topology. We also present some counterexamples to this statement for infinite discrete groups with extreme properties (HNN-group, Osin group, Ivanov group).

Paper reference: Preprint 46, Max-Planck-Institut fur Mathematik, 2005.

Categorified Associahedra and alternate blueprints for a free group element.

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The boundary of the nth associahedron K(n) is topologically equivalent to the $(n-3)$–sphere. The boundary of the nth composihedron $CK(n)$ is topologically equivalent to $S^(n-2)$. One way of describing the indexing of vertices of the composihedra is by referring to equivalent binary lists of words, as opposed to binary lists of generators as in the associahedra. Another way is to refer to binary trees with weighted leaves, where the weights of the leaves sum to n. This generalizes the indexing of the associahedra by binary trees with n leaves. This last point of view allows us to count the vertices of the composihedra by the binomial transform of the Catalan numbers. It also allows us to construct a realization of $CK(n)$ as a convex polytope, using methods recently developed by Loday for the associahedra. The new polytopes can be seen as a version of Stasheff's associahedra where sets have been replaced by objects in a general category.

When labeling vertices of the composihedra with bracketed lists of generators and words, we can view the nth polytope as representing an arbitrary reduced group element in the free group. Each vertex describes a sequence of concatenations for building that element from shorter ones. If the group is not free then the group element will instead be represented by a complex of polytopes, with faces identified in which the same rewriting of the same words has occurred between each vertex of those faces. The topology of such complexes is shown for some small examples.

High dimensional filling invariants of groups

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I will discuss a new method for bounding high dimensional Dehn functions. This is used to show that the spectrum of k-dimensional isoperimetric exponents of groups contains all rational numbers greater than or equal to $1 + 1/k$.

The asymptotic dimension of a curve graph is finite

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The asymptotic dimension, asdim, of a metric space was defined by Gromov as a quasi-isometric invariant. It was known that a delta-hyperbolic space with bounded geometry has finite asdim. An example is a word-hyperbolic group. We show that the curve graph of a compact surface has finite asdim. A curve graph is delta-hyperbolic but does not have bounded geometry.

Hanoi Tower Groups and Schreier Graphs

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We will associate with a famous puzzle known as Hanoi Tower Game on k-pegs a group which plays a role of the renorm group for this game. Some properties of Hanoi groups will be discussed including generation by a finite automaton and branching. A spectral properties of the Schreier graph in case of three pegs will be described as well.

Metric and growth properties of R.Thompson's group F

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We will give a review of results that concern metric and growth properties of R.Thompson's group F and its generalizations. We are going to discuss algorithms to find the length of a given element, to review recent progress in finding the growth functions of the groups and their positive monoids. Also we want to talk about the Hilbert space compression of F and mention some recent results that concern the amenability question, which remains open.

Coarse decompositions for boundaries of cubulated CAT(0) spaces

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In this talk we shall introduce a combinatorial notion of boundary $\Re H$ for a cubing $C(H)$ arising as the dual of a discrete ω-dimensional poc-set H. $\Re H$ has a natural median algebra structure, as well as a natural ordering whose intervals coincide with its intervals as a median algebra. Endowed with this structure, we call $\Re H$ the *Roller boundary* of the cubing $C(H)$.

When H happens to be a (discrete) G-invariant system of halfspaces in a CAT(0) space X endowed with a geometric action by a group G, we show how one can use $\Re H$ for producing a topologically meaningful decomposition/stratification of the CAT(0) boundary ∂X of X, having much to do with both the cone and Tits topologies on ∂X.

The sets into which ∂X is decomposed are defined as the fibers of a (discontinuous) map ρ of ∂X into $\Re H$; this map is actually well-defined for *any* reasonably discrete halfspace system in X, but has more interesting properties in the presence of a G-action.

Finally, we provide a criterion (in terms of the image of ρ) for G to act co-compactly on the cubing dual to H. This criterion links the co-compactness of the action of G on $C(H)$ with the quality of the approximation of boundary points by the halfspaces of H.

Our constructions and results provide a setting in which several issues of interest to geometric group theory are tied together: the end structure of semi-splittable groups, CAT(0) boundary topology, co-compact cubulations. In view of the results by Niblo-Reeves, Williams and Caprace, Coxeter groups supply a particularly good example of a setting to which this machinery can be applied in hopes of understanding their CAT(0) boundaries and the connections between boundary topology and, say, visual splittings of the corresponding group.

Nagata-Assouad dimension via Lipschitz extensions

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In the first part we show how to relate several dimension theories (asymptotic dimension with Higson property, asymptotic dimension of Gromov, and capacity dimension of Buyalo) to Nagata-Assouad dimension. This is done by applying two functors on the Lipschitz category of metric spaces: microscopic and macroscopic. In the second part we identify (among spaces of finite Nagata-Assouad dimension) spaces of Nagata-Assouad dimension at most n as those for which

the n-sphere S^n is a Lipschitz extensor. Large scale and small scale analogs of that result are given.

On balanced presentations of the trivial group

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To solve problems of Magnus and Stallings stated in the 1960s, we construct a balanced presentation of the trivial group such that no defining relator of this presentation could be replaced by a free generator and the group, given by the altered presentation, would still be trivial. Some other related problems and results on balanced presentations of the trivial group will be discussed.

Certain quotients of HNN-extensions which are HNN-extensions.

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Let G be a group with an HNN-presentation $P = \langle H, t \,\|\, t^{-}1Ut = V\rangle$. Let R be a cyclically reduced word in G and let N be its normal closure in G. We give a sufficient condition on R in order that G/N has an HNN presentation $Q = \langle K, t \,|\, t^{-}1Lt = M\rangle$,where $H < K$, $U < L$, $V < M$. We apply this to the case when G is a one-relator group and get a family of two relator groups with HNN-presentation.Under certain conditions this leads to a solution of the word problem and related problems.

Generalized triangle inequalities and their applications

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I will explain how to generalize the usual triangle inequalities and how to apply them to basic problems in linear algebra and theory of algebraic groups.

Orthogonal systems in graphs and free partially commutative groups

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Let Γ be a finite graph and G_Γ be the corresponding free partially commutative group. We construct orthogonality theory for graphs and free partially commutative groups and then use it to construct a nice theory of parabolic and quasiparabolic subgroups.

As applications we obtain a description of the centralizer of an arbitrary subset of G_Γ and compute the height of the lattice of centralizers of G_Γ.

Isomorphism and other algorithmic problems for fully residually free groups

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I will talk about the effectiveness of the JSJ decomposition for finitely generated fully residually free groups (joint result with A. Myasnikov), about the decidability of the isomorphism problem (joint result with I. Bumagin and A. Myasnikov) and about other algorithmic problems (joint results with D. Serbin)

One-relator relative presentations

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Adding two generators and one arbitrary relator to a nontrivial torsion-free group, we always obtain an SQ-universal group.

I shall discuss this and other properties of one-relator relative presentations.

Bigroups and a Limit Variety of Groups

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It follows easily from Zorn's lemma that if a variety of groups $\mathbf{V}$ is not finitely based then it contains a subvariety $\mathbf{V}^*$ such that all proper subvarieties of $\mathbf{V}^*$ are finitely based, but $\mathbf{V}^*$ itself is not. Any variety with these properties is called a *limit* or a *just non-finitely based* variety. In this sense limit varieties of groups form a "border" between those which are finitely based and those which are not. It is known that infinitely many such varieties exist (Newman, 1971) although no explicit examples are known.

In 2001 the first example of a limit variety $\mathbb{V}$ of bigroups was constructed by C.K.Gupta and the speaker. *A bigroup* is a pair (H, π) consisting of a group H and an idempotent endomorphism (projection) π of H. One can consider π as a unary operation on H so a bigroup is a universal algebra.

Let $\mathbf{U}$ be the variety of groups defined by

$$\mathbf{U} = \text{ var } \{H \mid \mathcal{H} = (H, \pi) \in \mathbb{V} \text{ for some projection } \pi : H \to H\}.$$

In other words, the variety of groups $\mathbf{U}$ is generated by all the bigroups $\mathcal{H} \in \mathbb{V}$ if we consider them as groups and "forget" about the additional operation π on $\mathcal{H}$. C.K.Gupta and the speaker conjectured that $\mathbf{U}$ is a limit variety of groups, that is, (i) $\mathbf{U}$ is a non-finitely based variety and (ii) each proper subvariety of $\mathbf{U}$ is finitely based. Our main result confirms the item (i) of the conjecture.

Theorem. *The variety of groups* $\mathbf{U}$ *described above is non-finitely based.*

On some infinite demensional linear groups

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The theory of finite dimensional linear groups is one of the best developed in Algebra. Because of the rich interplay between geometrical and algebraic ideas associated with finite dimensional linear groups they have played an important role in Group Theory. If the dimension is infinite, a situation is totally different. The study of the subgroups of infinite dimensional linear groups in this case is impossible without some essential additional restrictions. The series of the brilliant constructions developed by A. Yu. Olshanskii and his students is a very valuable argument supported this statement. The circumstances here are similar to those, which appeared in the early period of the development of Infinite Group Theory. One of the fruitful approaches there consisted from the application of finiteness conditions. The theory of finitary linear groups has

been successfully developed on this way. The subgroup G of $GL(F, A)$ (where F is a field, A is a vector space over F) is called finitary if for each element g from G the (subspace) centralizer of g in A has finite codimension in A. We consider another finiteness condition sporadically mentioned in some works but did not systematically studied yet. A subgroup G of $GL(F, A)$ is said to be the linear group with finite orbits if the set $aG = \{ag : g \in G\}$ is finite for every element a of A. Similarly, we can consider not only G-orbits of the elements, but G-orbits of the subspaces as well. A subgroup G of $GL(F, A)$ is said to be the linear group with finite orbits of subspaces if the set $\{Bg : g \in G\}$ is finite for every subspace B of A. In this case the index of the normalizer of B in G is finite for every subspace B of A. In particular, if the dimension of A in F is finite, then a linear group with finite orbits of subspaces is almost diagonal, thus it is abelian-by-finite. As the first step here we consider some classes of soluble linear groups with finite bounded orbits of elements (respectively finite bounded orbits of subspaces).

Homology identities of nilpotent groups have the finite basis property.

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We define a notion of homological identity and discuss the theorem given in the title.

On descriptions of canonical left cells in extended affine Weyl groups

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Let G be the general linear group defined over the complex numbers. In this paper we study descriptions of canonical left cells containing a unique element of minimal length in the double cosets of the Weyl group of G parametrized by the dominant weight of G

Growth of self-similar groups

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Let T_d is d-regular rooted tree and $G \leq AutT_d$. Let u be a vertex of T_d and $g \in G$. Let us denote by $\psi_u(g)$ the restriction of the action of the element g on the tree T_d to the subtree $T(u)$ with root u. Let us define $\psi_u(G) = \{\psi_u(g); g \in G\}$. A group G is called *self-similar* if for any vertex u $\psi_u(G) = G$ after the identification of the tree T_d with a subtree $T(u)$. We denote $T_d^{(k)} = \{u \in T_d\,;\, dist(u) = k\}$, where $dist(u)$ is a distance between u and the root vertex of the tree T_d. We say that a finitely generated self-similar group G is *ordinary* if there is system of generators S and a natural k such that

$$l_S(g) \leq \sum_{u \in T_d^{(k)}} l_S(\psi_u(g)),$$

where $l(*)$ is the length of an element $*$ with respect to the set of generators S. For an ordinary group G and for a natural number k we consider the *set without contracting*:

$$F_{G,S,k}(n) = \{g \in G \,|\, l_S(g) = \sum_{u \in T_d^{(k)}} l_S(\psi_u(g)) \leq n\}.$$

Denote $f_{G,S,k}(n) = |F_{G,S,k}(n)|$. A function $\gamma_{G,S}(n) = |\{g; l_S(g) \leq n\}|$ is called the *growth function* of the group G for the set of generators S. A non decreasing function $t(n)$ has a *subexponential growth* if

$$\lim_{n\to\infty} \sqrt[n]{t(n)} = 1.$$

Theorem. Let

$$\lim_{n\to\infty} \sqrt[n]{f_{G,S,k}(n)} = 1$$

for some natural k, then

$$\lim_{n\to\infty} \sqrt[n]{\gamma_{G,S}(n)} = 1.$$

This result allows us to prove that well-known p-groups of Gupta-Sidki and Gupta's group have intermediate growth.

Biautomaticity and Nonpositively Curved Piecewise Euclidean Complexes

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I will discuss a condition implying that a locally CAT(0) piecewise Euclidean 2-complex has biautomatic fundamental group. This result relies on a theorem relating paths in the 1-skeleton of the universal cover, and includes results by Gersten-Short and Niblo-Reeves (in dimension 2) as corollaries.

SO(n)-invariants of several matrices and quivers

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Suppose that a group $G < GL(n)$ acts on the direct sum $M(n)^d = M(n) \oplus \cdots \oplus M(n)$ of $n \times n$ matrices by the diagonal conjugation. This action induces the action of G on the coordinate ring $R = K[M(n)^d]$ in a natural way. Denote by R^G the algebra of invariants. We proved that over an infinite field of the characteristic different from 2 the algebra $R^{SO(n)}$ is generated by the pfaffians and the coefficients of the characteristic polynomial of products of the generic matrices and the transpose generic matrices. Similar result we also obtained for quivers. So the problem of describing generators of the algebra of invariants of a quiver is solved for a product of any classical groups, i.e., for $GL(n)$, $O(n)$, $Sp(n)$, $SL(n)$, $SO(n)$, where the characteristic of the base field is different from 2 wherever we talk about $O(n)$ and $SO(n)$.

Linear representations of the automorphism group of a free group

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Let F be a free group on $n > 2$ generators. The group $A = Aut(F)$ is a much studied group but very little seems to be known on its (finite dimensional) linear representations. We present a very rich collection of new representations which show that the representation theory of $Aut(F)$ is much richer than assumed before. By studying the action of suitable finite index subgroups of A of the relation module of carefully chosen finite groups we show that many interesting arithmetic groups appear as images of representations of A. Joint work with Fritz Grunewald.

Finite groups and hyperbolic manifolds

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The isometry group of a closed hyperbolic n-manifold is finite. We prove that for every $n > 1$ and every finite group G there is an n-dimensional closed hyperbolic manifold whose isometry group is G. This resolves a long standing problem whose low dimensional cases n=2 and n=3 were proved by Greenberg ('74) and Kojima ('88) resp. The proof is nonconstructive; it uses a 'probabilistic method', i.e. counting results from the theory of 'subgroup growth'. The talk won't assume any prior knowledge on the subject. Based on joint work with M. Belolipetsky (Inven. Math. Dec. 2005)

Olshanskii method and positive laws in groups

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There are many problems solved by means of Olshanskii geometric method. Some of these problems, concerning positive laws in groups, will be presented. The open problems will be formulated.

Path-components of Morse mappings spaces of surfaces

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Let M be a connected compact surface, P be either R^1 or S^1, and $F(M, P)$ be the space of Morse mappings $M \to P$ with compact-open topology. The classification of path-components of $F(M, P)$ was independently obtained by S. V. Matveev and V. V. Sharko for the case $P = R^1$, and by the author for orientable surfaces and $P = S^1$. We present an independent proof of this classification based on the structure of the mapping class group of a surface. The main observation is that "elementary" diffeomorphisms like Dehn twists and Y-diffeomorphisms that generate mapping class group preserves some Morse function. Our approach has a close relationship to the representation of the mapping class group obtained by A.Hatcher and W.Thurston via Morse functions.

Kazhadn constant for automorphisms of nilpotent groups
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Let $G = F(k, c)$ be the free nilpotent group of class c on k generators. Luboztky and Pak have shown that the group of the automorphisms of G which is generated by the set S consisting of Nielsen transformations has the Kazhdan property T. In this talk we propose a method for computing a lower bound for the Kazhdan constants with respect to S.

Combinatorial horoballs, quasi-geodesic bicombings, and relative hyperbolicity
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Relatively hyperbolic groups were first defined by Gromov. Since then, many equivalent characterizations have been given. I will discuss a new characterization of relatively hyperbolic groups which is close to Gromov's original definition, but made concrete in such a way that combinatorial tools originally developed for hyperbolic groups can be adapted to relatively hyperbolic groups. If time permits, applications to "Dehn filling" of relatively hyperbolic groups will also be discussed. This is joint work with Daniel Groves.

Algorithmic problems in amalgams of finite groups
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It turns out that finitely generated subgroups of amalgams of finite groups can be effectively represented by finite cannonical graphs. These graphs posses all the essensial information about the subgroups, which enables one to use them in order to solve various algorithmic problems: the membership problem, the finite index problem, the conjugacy of subgroups, the freeness problem, the separability problem, the reading of Kurosh decomposition (in the case of free products) and others.

We'll present some problems of the above list and their solutions.

From idempotent-generated semigroups to 2-complexes
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In 1973 Nambooripad introduced the notion of a biordered set as an axiomatic characterization of the set of idempotents of a semigroup relative to certain basic products. By associating a 2-complex with a biorderd set in a natural way, we are able to prove that every finitely presented group arises as a maximal subgroup of the free idempotent-generated semigroup on a finite biordered set, thus disproving a conjecture of Easdown about the structure of these groups. It follows that the word problem for the free idempotent-generated semigroup on a finite biordered set is undecidable.

On elementary theories of free groups
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In this talk I am going to discuss some classes of groups and several new techniques that play a key part in recent solution of the Tarski's problems on elementary theories of free groups.

Decompositions of Coxeter groups over minimal splittings
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The first "accessibility" questions for finitely generated groups arose from Stallings' splitting theorem for infinite ended groups. Later Dunwoody proved that finitely presented groups are indeed accessible with respect to splittings over finite groups. Bestvina and Feighn followed Dunwoody's result with an accessibility result for finitely presented groups when splittings subgroups are "small". The notion of JSJ-decompositions of finitely presented groups arose from geometrical/algebraic decompositions of closed 3-manifolds over certain embedded separating surfaces. For 1-ended finitely presented groups Rips and Sela showed the existence of (unique) JSJ decompositions over 2-ended splittings. Dunwoody and Sageev introduced the notion of minimal virtually abelian splitting subgroups into the JSJ arena.

We introduce a notion of "strong accessibility" over minimal splittings of groups that naturally generalizes the original accessibility results over finite and

small splitting subgroups. We discuss two results. The first is a strong accessibility result for finitely generated Coxeter groups over minimal splittings. The second is a "best possible" JSJ-result for splitting finitely generated Coxeter groups over virtually abelian splitting subgroups. Splittings over minimal virtually abelian subgroups plays an important role in the JSJ result. Both results are "visual" in the sense that the critical decompositions involved can be "seen" geometrically in the presentation diagram of a Coxeter group.

Continuum varieties of groups and verbal embeddings of groups

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In this talk we would like to show a few applications of the argument developed by Ol'shanskii for construction of continuum sets of varieties of groups to the concept of verbal embeddings of groups. The embedding φ of the group H into the group G is *verbal* for the given word set $V \subseteq F_\infty$ if the isomorphic copy $\varphi(H)$ lies in the verbal subgroup $V(G)$. Using appropriately chosen groups H with appropriate word sets V we build continuum sets of soluble 3-generator non-Hopfian groups which are non-isomorphic and, moreover, generate pairwise distinct varieties of groups.

On the engulfing property for word hyperbolic groups

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A group G engulfs its subgroup H, if there exists a proper finite index subgroup $K \leq G$ such that $H \leq K$. We will say that a hyperbolic group has the Engulfing Property if it engulfs every proper quasiconvex subgroup.

Let G be a group satisfying the Engulfing Property. We prove that each quasiconvex subgroup $H \leq G$ has a finite index in its profinite closure H^* in G. If, in addition, G is residually finite (or torsion-free) we show that $H^* = H$. In particular, this generalizes results of D. Long and G. Niblo-B. Williams.

Relative hyperbolicity and bounded cohomology
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1. Several characterizations of relatively hyperbolic groups will be given in terms of relative bounded cohomology.
2. It is certainly known to the participants of this conference that proving anything about relatively hyperbolic groups requires a lot of time and space. I will attempt presenting a language that streamlines statements about relatively hyperbolic groups. We will discuss angles, snake metrics, fine triangles, tuples, and ideal complexes.

Dimension zero at all scales
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We consider the notion of dimension in two categories: the category of separable metric spaces and Lipschitz maps, and the category of separable metric spaces and uniform maps. A uniform treatment is given to the large scale dimension and the small scale dimension. We show that in both categories a space has dimension zero if and only if it is equivalent to an ultrametric space. There is a universal zero-dimensional space in both categories. Spaces of dimension zero are characterized by means of extensions of maps to the unit 0-sphere and by means of retractions to subspaces. Any countable group of asymptotic dimension zero is coarsely equivalent to a direct sum of cyclic groups. We construct uncountably many examples of coarsely inequivalent ultrametric spaces.

Splitting and CAT(0) superrigidity for lattices in products
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CAT(0) spaces generalize notably symmetric spaces and Bruhat-Tits buildings. Therefore, one can reformulate Margulis' superrigidity theorem in terms of actions on CAT(0) spaces. We propose such a generalization for irreducible lattices in products. The proof uses a general Lawson-Yau/Gromoll-Wolf splitting theorem.

Infinite simple groups of large commutator width

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In 1977, I. M. Isaacs asked if there exists a non-abelian simple group containing an element which is not a commutator. In terms of commutator width, the question is whether there exist simple groups of commutator width greater than 1.

We show how to construct simple groups of arbitrarily large commutator width, presenting such groups by generators and defining relations.

Simply connected subsets of buildings

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Buildings have been introduced by Tits in order to study groups of Lie-type. There are essentially two points of view. One can see buildings as simplicial complexes or as chamber systems. In my talk I will choose the second one. Chamber systems arise naturally as a sort of generalized Cayley graphs and it is therefore not surprising, that they are quite useful for investigating presentations of groups.

In my talk I will present a criterion for the simple connectivity of a subset of a simply connected chamber system which is based on filtrations. I will present two applications to group theory, one of which is turns out to be very useful in the classification of the finite simple groups

A minimal Cantor set in the space of 3-generated groups

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I will describe a family of just-infinite branch 3-generated groups parametrized by infinite binary sequences, which originates from a problem in holomorphic dynamics. Two groups in this family are isomorphic if and only if the corresponding sequences are cofinal. No two groups of the family can be distinguished by finite sets of relations. Other interesting properties of the groups will be discussed.

Obstructions to splitting

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Stallings' theorem asserts that a group G splits over a finite subgroup as a non-trivial amalgamated free product or HNN extension if and only if the pair $(G, \{e\})$ has more than one end. We examine obstructions to splitting a finitely generated group over an arbitrary subgroup or class of subgroups using a recent adaptation of Sageev's cube complex associated to a group pair (G, H) with more than one end. This yields some splitting theorems which generalise results from low dimensional topology to the class of all finitely generated groups.

Van Kampen diagrams, machines, and asymptotic behavior of groups

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Mostly I will speak on the results obtained jointly with M.V.Sapir after I moved to the Vanderbilt in 1999.

Consider a finite presentation of a group G in terms of generators and relators: $< A, R >$. Then for every word w in the group alphabet A, vanishing in G, there is a planar van Kampen diagram responsible for some deduction of the equality $w = 1$ from the defining relations $r = 1$ where $r \in R$. The Dehn function $d(n)$ gives the upper bound of areas of (minimal) diagrams whose perimeters $\leq n$. Up to a natural equivalence, it does not depend on the choice of the finite presentation for G. The asymptotic behavior of $f(n)$ is an important invariant of a finitely presented group G, connected to geometric and algorithmic properties of G.

It is not difficult to prove that every Dehn function is a time function of a Turing machine, but, unfortunately, the converse claim is false, and to investigate Dehn functions one is to work hard and discover new properties of diagrams and new types of them. Our method presents answers to a number of long-standing problems in Group Theory,in particular, on the complexity of computations in groups, on the algorithmic conjugacy problem, and on a problem about amenability of finitely presented groups. Our recent papers present examples of groups with strange behavior of their Dehn functions. These results are also applicable to the theory of asymptotic cones of groups.

Embedding theorems for countable groups and relative hyperbolicity
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I will give a brief and elementary survey of the small cancellation theory over relatively hyperbolic groups and discuss some applications. In particular, we will establish a uniform approach to proving embedding theorems for countable groups. Our method allows to obtain many new results as well as to reprove some well-known theorems originally proved by Alexander Olshanskii and his students.

Cut points and splittings of CAT(0) groups
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We show how to associate to a continuum an R-tree which encodes all pairs of points that separate the continuum. We apply this in the case of CAT(0) groups and we show that if a pair of points separates the boundary of a CAT(0) group then the group splits over a 2-ended group. We discuss how can one see the JSJ decomposition of a CAT(0) group from its boundary.

Finiteness properties for the kernel of the pure motion group of n unlinked loops
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The pure symmetric automorphism group of a free group equals the pure motion group of n unlinked loops in R^3 and contains the pure braid group as a subgroup. The homomorphism between these loop groups induced by forgetting a loop extends the homomorphism between pure braid groups induced by forgetting a puncture. We study the homology of the kernel of the loop homomorphism via its action on a certain subcomplex of Autre Space.

Some Tarskiis-type problems and logical invariants of algebras

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Let Θ be a variety of algebras and G be and algebra from Θ. Denote by $LK_\Theta(G)$ the category of elementary sets, i.e., the sets defined by First Order formulas. This category is a logical invariant of the algebra G. We are interested in the situation when the categories $LK_\Theta(G_1)$ and $LK_\Theta(G_2)$ are isomorphic. We will define the notion of strong elementary equivalence of algebras. It can be seen that if the algebras G_1 and G_2 are strongly elementary equivalent then they are elementary equivalent. But the converse statement is not true. Besides, strong elementary equivalence implies geometric equivalence of algebras and an isomorphism of the corresponding categories.

Among the numerous arising problems we will distinguish the following Tarskiis-type problem:

Let F_n and F_m be two noncommutative free groups with n and m generators, respectively. Now it is known that they are elementary equivalent. Is it true that they are strongly elementary equivalent?

This problem is associated with the following question: is it true that every free group is logically noetherian? In other words this is a question about a generalization of Gubas theorem for free groups.

All questions above relies on a special approach to algebraic logic.

How to define the Burnside-type problems for solvability property of groups and Lie algebras

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In the talk we discuss the ways for posting Burnside-type problems related to solvability property. The classical Engel-Zorn-Baer results connect Engel and nilpotency properties of groups and Lie algebras. We describe the obtained counterparts of the Engel-Zorn-Baer results in the solvable case. These results constitute a ground for the corresponding Burnside-type problems.

Matching Theorems for Systems of a Finitely Generated Coxeter Group

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We prove a series of matching theorems for two sets of Coxeter generators of a finitely generated Coxeter group that identify common features of the two sets of generators. As an application, we describe an algorithm for finding a set of Coxeter generators of maximum rank for a finitely generated Coxeter group.

We prove that any two Coxeter systems of maximum rank for a finitely generated Coxeter group have the same number of visual subgroups of each complete system isomorphism type; in other words, the presentation diagrams of two Coxeter systems of maximum rank for a finitely generated Coxeter group have the same number of complete subdiagrams of each isomorphism type.

Pro-p groups with non-isomorphic discrete and continuous cohomology groups

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We construct examples of finitely generated pro-p groups with non-isomorphic discrete and continuous cohomology groups.

Geometric notions of space complexity for the word problem

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A word w represents the identity element in a finitely presented group if and only if it can be reduced to the empty word by applying relations and inserting or deleting inverse pairs of letters. The minimal length $FL(n)$ such that every word w of length at most n that represents the identity can be reduced to the empty word whilst encountering words no longer than $FL(n)$ en route, is a (naive) measure of the space complexity of the word problem. This function is called filling length and has a geometric interpretation in terms of null-homotopies of loops; its qualitative behaviour gives a quasi-isometry invariant. I will show that relaxing the definition in natural ways (such as allowing conjugation when reducing) can radically change the invariant.

Free product decompositions in images of certain free products of groups

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In 1978 the speaker proved the following result:

Let G be a group which has a presentation with n generators $x_1, \ldots, x_n$ and m relators, where $m < n$, and let $S = \{x_1, \ldots, x_n\}$. Then some subset of $n - m$ elements of S freely generates a free group.

The history of this result dates back to 1930, when Magnus published his Freiheitssatz, which is essentially the case of our statement in which $m = 1$. In 2004 J.S.Wilson generalized above-mentioned result by proving a similar statement in which S is any generating set for G. The proof was indirect, relying on another result of the speaker.

Here we give a direct proof of a considerably more general result. Roughly speaking, the improvement consists of the replacement of the elements x_i by subgroups, of the members of S by suitably small subgroups, and of the hypothesis that S generates G by a weaker hypothesis.

On rigidity and the isomorphism problem for four strand tree braid groups

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Given a tree braid group B_nT on $n = 4$ strands, we are able to reconstruct the tree T. Thus tree braid groups B_4T and trees T (up to homeomorphism) are in bijective correspondence. That such a bijection exists is not true for higher dimensional spaces, and is an artifact of the 1-dimensionality of trees. We use this bijection to solve a version of the isomorphism problem for tree braid groups with $n = 4$ strands. We also make some comments on the possibility of generalizing this solution to tree braid groups with more strands.

Groups acting on tree-graded spaces

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We develop a theory of groups acting on tree-graded spaces generalizing the Rips-Bestvina-Feighn-Sela theory of groups acting on $\mathbb{R}$-trees. Tree-graded spaces appear as asymptotic cones of a wide class of groups including mapping class groups, fundamental groups of graph manifolds, geometrically finite Kleinian groups and relatively hyperbolic groups in general. Our results allow us to describe relatively hyperbolic groups G with infinite $Out(G)$ (generalizing results of Rips-Sela's and others), and establish results about splittings of groups with infinitely many homomorphisms into a relatively hyperbolic groups.

Random Quotients of the Modular Group are Rigid

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We prove that quotients of the modular group, by any finite number of additional relators, generically satisfy a very strong Mostow-type rigidity. The associated geometric structure of such a quotient is its Cayley graph on standard generators, a and b, of the modular group with the word metric. Generically, two quotients are isomorphic if and only if their associated Cayley graphs are isometric. Indeed, one can at most interchange the edge label b and $b^(-1)$.

As a consequence, although the isomorphism problem remains unsolvable for quotients of the modular group, it generic-case complexity is strongly polynomial time. Random quotients are "essentially incompressible". This means that the shortest possible finite presentation of such quotients are uniformly "almost" as long as the given presentation. Also, one can calculate the exact asymptotics as n goes to infinity of the number of isomorphism classes of quotients with m relators all of length n.

Groups with regular free length functions in Z^n

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Finding description of finitely generated groups acting freely on Λ-trees, or, equivalently groups having free length functions in Λ (where Λ is an ordered abelian group) is one of the major problems in Geometric Group Theory. This problem was solved for some special cases of Λ but still is far from being solved in the general case. In my talk, at first, I'm going to introduce a natural restriction on length function which is called the regularity condition. Then I'm going to present the description of finitely generated groups with regular free length functions in Z^n, and to discuss why regularity condition is important.

On the growth in semigroup varieties

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Let S be a semigroup and let $M = varS$ be the semigroup variety generated by S. We study the problem: what type of growth can finitely generated M-semigroups have. Some cases where every finitely generated M-semigroup has polynomial growth will be considered.

Translation equivalence in free groups

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Motivated by the work on hyperbolic equivalence of homotopy classes of closed curves on surfaces, we investigate a similar phenomenon for free groups. Namely, we study the situation where two elements g, h in a free group F have the property that for every free isometric action of F on an $\mathbb{R}$-tree X the translation lengths of g and h on X are equal (or have bounded ratio). This is equivalent to the following combinatorial property: for any automorphism φ of F, the cyclic lengths of $\varphi(g)$ and $\varphi(h)$ are equal (or have bounded ratio).

Uniform Kazhdan Groups

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For a discrete group G and a finite subset X of G, let K(G, X) be the Kazhdan constant of G with respect to X. We define the uniform Kazhdan constant of G by

$$K(G) = \min\{K(G, X) \mid X \text{ is finite and generates } G\}.$$

We provide an example of an infinite group with non - zero uniform Kazhdan constant.

Algebraic combinatorics, semigroup representations and random walks on hyperplane chambers after Ken Brown

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Ken Brown observed that the eigenvalues with multiplicity for certain random walks on the chambers of a hyperplane arrangement could be calculated using the representation theory of finite semigroups and Rota's theory of Möbius inversion. The underlying idea is the same as Diaconis' approach to random walks on finite abelian groups: both rely on the fact that the semigroup algebras in question are triangularizable and so the eigenvalues of the Markov operator are certain character sums.

Inspired by this we have developed algebraic-combinatorial tools to compute multiplicities of irreducible representations for a large class of finite semigroups including inverse semigroups, semigroups with commuting idempotents and semigroups with triangularizable algebras. In particular eigenvalues can be calculated (with multiplicity), via character sums, for random walks on finite semigroups with abelian subgroups whose von Neumann regular elements satisfy an identity of the form $x^m = x$.

Hanoi Towers group as iterated monodromy group
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We present the Hanoi group on 3 pegs as iterated monodromy group of a rational post-critical function on the Riemann Sphere. We also consider several related groups and their associated rational functions and Julia sets. As an application, we show that all Hanoi groups have exponential growth. We also show that the Hanoi group on 3 pegs is a finitely generated branch group that is not just infinite.

Automorphism groups of trees and Sylow p-subgroups of finitary symmetric groups
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In the talk we will survey some results concerning the structure of automorphism groups of special ended trees, and we present a full description in these terms of Sylow p-subgroups of finitary symmetric groups over infinite sets.

The ephemeral Morita classes
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There is a series of cocycles in the cohomology of the group of automorphisms of a free group (first discovered by S. Morita) which have an elementary description in terms of finite graphs. The first two of these give nontrivial cohomology classes, and it is conjectured that they are all nontrivial; on the other hand according to a recent result of S. Galatius all classes must be stably trivial. We show that a single application of the stabilization map $Aut(F_n)$ to $Aut(F_{n+1})$ kills these classes, so that they in fact disappear immediately after they appear.

Spaces of Translations

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The simplest nontrivial dynamical systems are arguably irrational rotation on the circle. In this talk, I will try to generalize their classification to other (almost as simple) translation systems.

Sylow Objects in Finite Groups and the Factorization of Formations

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It is well known that the classic Sylow theorem is the most important result of groups and has numerous applications. In particular, we should mention that Sylow objects such as p-subgroups and their normalizers have played an important role in the problems of classification of finite simple groups.

Within the framework of the theory of formations, Sylow objects also played an important role. Remember that if a finite group G belongs to a saturated formation $\mathfrak{F}$ and G has a composition factor of order p, then the class $\mathfrak{N}_p$ of all finite p-groups is contained in $\mathfrak{F}$. Analogy to Sylow subgroups is seen here, therefore the formations of the $\mathfrak{N}_p$ type can be called Sylow objects in the theory of formations.

In this report, we give a brief introduction on some of the new research along the two directions.

Special Cube Complexes

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We identify a class of "special" nonpositively curved cube complexes that are closely related to right-angled Artin groups. We give applications to subgroup separability and linearity, and to Coxeter groups. Some sample consequence of our theory include:

1) Every word-hyperbolic Coxeter group has separable quasiconvex subgroups.

2) Let G be the group given by the HNN extension $\langle a, b, t \,|\, U^t = V\rangle$. Then G is a subgroup of $SL_n(Z)$ unless U and V have conjugate powers.

3) For each f.p. group Q, there is a short exact sequence $1 \to N \to G \to Q \to 1$ where $G < SL_n(Z)$ and N is f.g.

Uniform spaces and Gromov hyperbolicity

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We establish a connection between Gromov hyperbolic spaces and quasiconformal analysis: We characterize uniform spaces in terms of Gromov hyperbolic spaces and the quasiconformal structure on the Gromov boundary. We use a notion of inversions in general metric spaces. As an application, we show that a quasimobius map between domains in metric spaces with annulus quasiconvexity sends a uniform domain to a uniform domain.

Scaled relators and Dehn functions for nilpotent groups

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A homogeneous nilpotent Lie group has a scaling automorphism determined by a grading of its Lie algebra. Many proofs of upper bounds for the Dehn function of such a group depend on being able to fill curves with discs compatible with this grading; the area of such discs changes predictably under the scaling automorphism. We will present combinatorial methods for finding such bounds. Applications include constructing the first example of a torsion-free nilpotent group of class 3 with a quadratic Dehn function.

Limits of Thompson's group F

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Denote by $\mathbb{F}$ the Thompson's group F with standard presentation: $\mathbb{F} = \langle x_0, x_1 | [x_0 x_1^{-1}, x_0^{-1} x_1 x_0], [x_0 x_1^{-1}, x_0^{-2} x_1 x_0^2] \rangle$ and fix a sequence $\{g_i\}_{i \in \mathbb{N}}$, where $g_i \in \mathbb{F}$ for all i. Let $G_i = \langle x_0, x_1, x_2 | [x_0 x_1^{-1}, x_0^{-1} x_1 x_0], [x_0 x_1^{-1}, x_0^{-2} x_1 x_0^2], x_2^{-1} g_i \rangle$ be a sequence of isomorphic copies of $\mathbb{F}$ marked by three elements. We investigate the convergence of such sequences and possible limit groups constructed in this manner. It is easy to see, that at infinity we can get free and direct products of $\mathbb{F}$ with $\mathbb{Z}$. We study free constructions involving $\mathbb{F}$ and $\mathbb{Z}$ which can be obtained by this procedure. In particular, we prove that no (centralized) HNN-extensions occur as $\mathbb{F}$-limit group.

Some open problems in Ring Theory

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We will review some old and new open problems in the theory of algebras and their connections to Combinatorial Group Theory.