Read about lower and upper central series of a group in the book.

0. (***) Prove that every group of order 60 with more than 1 Sylow 5-subgroup is simple.

1. (**) Show that $UT(n, \mathbb{R})$ is a nilpotent group. Find its upper and lower central series.

2. (a) (**) Find the upper central series of the group $GL(n, \mathbb{R})$.
   (b) (**) Find the lower central series of that group.

3. (a)(**) Find the upper central series of the group $Iso(\mathbb{R}^2)$
   b) (**) Find the lower central series of that group.

4. (**) Let $p$ be the smallest prime dividing the order of a finite group $G$. Prove that every subgroup of $G$ of index $p$ is normal.

5. (**) Show that any group of order 132 is not simple.

6. (**) Show that any group of order 312 contains a normal Sylow subgroup.

7. (***) Show that, in a nilpotent group $G$, every normal subgroup nontrivially intersects the center $Z(G)$. 