SYLLABUS FOR MATH 9800 - MODULAR FORMS
Vanderbilt University, Fall 2020

Course Description and Objectives

According to Martin Eichler, there are five fundamental operations of mathematics: addition, subtraction, multiplication, division, and modular forms. In this course, students will learn how to justify this claim, and in particular will see many of the beautiful applications of modular forms to number theory and other areas of mathematics. For example, modular forms are central to the proof of Fermat's Last Theorem, and can be used to show other Diophantine results, such as the fact that 144 is the largest Fibonacci number which is also a perfect power. Modular forms have a knack for showing up in surprisingly deep proofs of very simple-to-state results like these, and of many surprising facts, such as the seemingly innocuous (but very deep) observation that $e^{\pi\sqrt{163}} = 262537412640768743.99999999999925\ldots$ is incredibly close to being an integer. These applications continue to arise in hot-topic areas of mathematics; in fact, modular forms proofs of cases of the sphere packing problem, which asks for optimal arrangements of spheres to fill up as much space as possible (think stacking of oranges in a grocery store), have led to a flurry of activity just in the past few years. Roughly speaking, modular forms are complex functions which are periodic, like sine or cosine, but satisfy infinitely many more symmetries simultaneously. This may seem surprising at first, and satisfying infinitely many symmetry properties is indeed very constraining. In fact, it allows one to use basic complex analysis to build up a very rigid algebraic theory of these functions. This is also what makes modular forms so special and where applications to arithmetic and number theory arise; for example, these symmetries turn them into a tool for proving infinitely many identities with a finite computer check. In this course, we will survey this theory and its applications, as well as its connections to other objects of number theory such as elliptic curves and elliptic functions, with an eye towards understanding Tunnell's criterion determining which integers n are congruent (that is, areas of right triangles with rational side lengths... try to see if you are able to determine a few examples for yourself!).

Lectures Mondays, Wednesdays, and Fridays from 12:40-1:30 in Buttrick 206

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Suggested Textbooks

N. Koblitz: Elliptic curves and modular forms
T. Apostol: Modular Functions and Dirichlet series in number theory

Prerequisites No knowledge of elementary number theory is assumed, however it would be recommended that students are familiar with applying the main theorems of complex analysis (Identity theorem, Cauchy's theorem, Residue theorem, etc.). However, the basic uses of these theorems can also be learned concurrently, depending on the background of the class.

Course Website `https://math.vanderbilt.edu/rolenl/ModularForms.html` Brightspace will also be used for announcements and discussions, as well as linked Zoom sessions if you need to attend remotely or the course transitions to online.

Office hours times Tuesdays from 1:00-3:00 and Thursdays from 11:00-12:00. Office hours will be held virtually on Zoom.

Grading Along with the lectures, there may occasionally be optional homework problems. In addition, regular attendance is expected. Due by December 4, you are expected to turn in a final project.

This will be an expository 5-7 page account of a topic which is related to, but not directly covered in, the lectures. A good example would be a further account of applications to the theory of partitions, the use of modular forms in proving recent theorems on sphere packings, or an exposition showing mastery of a by completing a paper using modular forms to prove some result which is interesting to you. You must have your topic approved by me, and I encourage doing so within the first month of class. You can discuss this project or advanced topic you'd like to learn about, or ideas for project suggestions, with me or any of your classmates as much as you want.

Academic Integrity Your work in this class is governed by Vanderbilt University's Honor Code. Work on the written project must be your own. If you are uncertain about how the Honor Code applies, please ask the instructor for clarification.

Accommodation procedure If you believe you may require special accommodations for a condition that may impact your work in this course, please contact the Equal Opportunity, Affirmative Action and Disability Services Department (`http://www.vanderbilt.edu/ead/`). The EAD will determine what accommodations are appropriate and communicate them to the instructor. This service is confidential.

Classroom Decorum Please arrive to class on time every day to avoid disturbing the learning environment. If you are late, please enter quietly and quickly. The use of electronic devices, other than for note-taking, is not allowed. If you are watching remotely on Zoom, please be actively engaged in the lecture, and have the microphone off except when asking a question to limit background noise or feedback. Sessions may be recorded and will be used only for educational purposes. Any student who records class without instructor permission and/or publishes recordings outside the Vanderbilt ecosystem is guilty of an Honor Code violation.

Important semester dates (for undergraduates only) The Open Enrollment Period ends on Monday, August 31st. This is the deadline for students to add a course or to make other changes in YES. Between September 1st and September 7th, any withdrawals or adjustments in level or in grading status must be completed using the add/drop form. If only the "DROP" section of the form is filled out, the instructor may "sign" the form. If a student wishes to make any change that involves filling in the "ADD" section of a drop/add form (whether or not it also involves filling in the "DROP" section), then the student must contact the DUS (John Rafter) or the Assistant DUS (Jakayla Robbins). Per Math Department policy, the only change to a math course that will be approved is a change to the level of the course (e.g. switching from Math 1301 to Math 1300 or vice versa).

Final remarks If you have any questions or concerns about anything in the class, please do not hesitate to come and talk to me at anytime. Feedback from students is very valuable to me and I am open to adjusting the course based on such feedback.