HOMEWORK 7

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due at the beginning of class on **Thursday, November 24**. Please put your name and course on your assignment, and make sure to staple your papers.

(1) Consider the basis $\{(-2,3,1),(3,-1,1),(1,-1,-1)\}$ of $\mathbb{R}^3$. Compute the coordinates of $v=(6,-2,1)$ with respect to this basis.

(2) Consider the function $T\colon M_{2\times 3}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ given by

$$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11}-a_{12} & a_{13}+2a_{12} \\ 0 & 0 \end{pmatrix}.$$

  (a) Show that T is a linear transformation.
  (b) Find a basis for $\ker(T)$.
  (c) Find a basis for $\operatorname{Im}(T)$.
  (d) What does the rank-nullity theorem claim in this case? Check that this indeed holds, using your answers from (b) and (c).

(3) Given linear transformations $T_1\colon V \to W$ and $T_2\colon W \to W'$ for vector spaces V, W, W', their composition $T = T_2T_1\colon V \to W'$ is their composition as functions. That is, if $v \in V$, then $T(v) = T_2(T_1(v)) \in W'$. Show that the composition T is also a linear transformation.

(4) We have seen that the subset of matrices in $M_{n\times n}(\mathbb{R})$ with trace zero (i.e., the sum of elements on their diagonals are zero) are a subspace of $M_{n\times n}(\mathbb{R})$. One way to find the dimension, as you did in a specific case on the last homework, is to explicitly write down a basis. However, there is another method, which this problem will guide you through.
  (a) Show that the function $\operatorname{tr}\colon M_{n\times n}(\mathbb{R}) \to \mathbb{R}$ which takes the trace of a matrix is a linear transformation.
  (b) Describe the kernel and image of this transformation.
  (c) Find the dimension of $\operatorname{Im}(\operatorname{tr})$.
  (d) Using the rank-nullity theorem, find the dimension of the subspace of trace zero matrices in $M_{n\times n}(\mathbb{R})$.