HOMEWORK 6

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due at the beginning of class on **Thursday, November 17**. Please put your name and course on your assignment, and make sure to staple your papers.

(1) Determine which of the following sets of vectors are linearly independent in $\mathbb{R}^4$.

(a)
$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \\ -1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 3 \\ 4 \\ -4 \\ 3 \end{pmatrix}.$$

(b)
$$v_1 = \begin{pmatrix} -1 \\ 2 \\ 3 \\ 5 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 7 \\ 100 \\ -10 \\ 11 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -\pi \\ 0 \\ 0 \\ 10 \end{pmatrix}, \quad v_4 = \begin{pmatrix} e \\ 0 \\ \pi^e \\ 11 \end{pmatrix}, \quad v_5 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

(c)
$$v_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \qquad v_4 = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}.$$

(2) Find a basis of the space of 2×2 real matrices with trace zero, i.e., those matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$ with $a + d = 0$.

(3) Suppose that $\{v_1, \dots v_k\}$ is a set of vectors in $\mathbb{R}^n$ and A is a matrix $A \in M_{m\times n}$. Further suppose that $\{Av_1, \dots Av_k\}$ is a linearly independent set of vectors in $\mathbb{R}^m$. Show that the original set of vectors $\{v_1, \dots, v_k\}$ is a linearly independent set of vectors in $\mathbb{R}^n$.

(4) Find bases for the kernel (null space), row space, and column space of the matrix
$$A = \begin{pmatrix} 0 & 2 & 8 & -7 \\ 2 & -2 & 4 & 0 \\ -3 & 4 & -2 & 5 \end{pmatrix}.$$