HOMEWORK 3

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due at the beginning of class on **Thursday, October 20**. Please put your name and student number, as well as your subject (Maths., TP, or TSM) on the back of your assignment, and make sure to staple your papers.

(1) Suppose that

$$A = \begin{pmatrix} 4 & 0 \\ 2 & 0 \\ 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \end{pmatrix}.$$

(a) Compute AB and BA.
(b) Compute, or show that it doesn't exist, the following inverses: $(AB)^{-1}$ and $(BA)^{-1}$.

(2) For any 2×2 matrix $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$, define the trace of A, denoted $\operatorname{tr}(A)$, as the sum of diagonal elements, i.e. $\operatorname{tr}(A) = a + d$. Further let A^2 be the product $A \cdot A$ for any square matrix A. Show that for any 2×2 matrix, we have

$$2 \det A = \det \begin{pmatrix} \operatorname{tr}(A) & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr}(A) \end{pmatrix}.$$

Hint: you may use the formula

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

(3) Consider the permutations

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 4 & 1 & 5 & 2 & 6 \end{pmatrix}, \qquad \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 1 & 7 & 5 & 6 \end{pmatrix}.$$

(a) Compute $\pi\sigma$.
(b) Write $\pi\sigma$ as a product of disjoint cycles.
(c) Use your answer from the last part to write $\pi\sigma$ as a product of transpositions.
(d) Use you answer from the last part to find $\operatorname{sign}(\pi\sigma)$.

(4) Compute $\det A$ **directly from the definition** (that is, as the unique alternating multilinear function on rows which has $\det I_n = 1$), where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 7 & 0 & 9 \end{pmatrix}.$$