HOMEWORK 1

MA1111: LINEAR ALGEBRA I, MICHAELMAS 2016

Solutions are due at the beginning of class on **Thursday, October 6**. Also attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. Please put your name and student number, as well as your subject (Maths., TP, or TSM) on the back of your assignment, and make sure to staple your papers. In general, work must always be shown to get full credit.

(1) Find the area of the triangle in the plane with vertices specified by their $3D$-coordinates $A = (1, 2, 0)$, $B = (3, 0, 0)$, and $C = (5, 4, 0)$. (Hint: this triangle constitutes half of a parallelogram.) You must use a result we learned from class, and cannot use assume other geometric results such as Heron's formula!

(2) Show (i.e., prove) that if v, w, w' are any three-dimensional vectors, then

$$v \times (w + w') = v \times w + v \times w'.$$

(3) In 4-dimensional (Euclidean) space $\mathbb{R}^4$, consider the "standard basis vectors" (these are the analogues of i, j, k in three-dimensional space) defined by

$$e_1 = (1, 0, 0, 0), \quad e_2 = (0, 1, 0, 0), \quad e_3 = (0, 0, 1, 0), \quad e_4 = (0, 0, 0, 1).$$

Suppose that f is a **multilinear** function $f(x, y, z)$ taking inputs of 4-d vectors x, y, z and outputting a real number. Suppose that the value of f on the standard basis vectors is determined by the equation

$$f(e_i, e_j, e_k) = i \cdot j - k$$

for $i, j, k = 1, 2, 3, 4$. For example, $f(e_2, e_3, e_4) = 2 \cdot 3 - 4 = 2$. Then find the value $f((8, 1, 3, 2), (5, 0, 7, 0), (0, 0, 0, 1))$.

(4) (a). Find the equation of the plane in $\mathbb{R}^3$ passing through the points $(1, 1, 6)$, $(3, 4, 2)$, and $(5, 4, 1)$.

(b). We define the angle between two planes as the angle between their normal vectors. Find the angle between the plane in part (a) and the plane given by the equation

$$2x + 3y + 6z = 10.$$

(You can leave your answer in terms of standard functions from your trigonometry class; i.e., a numerical approximation isn't needed).