Math 2106-D, Foundations of Mathematical Proof
SAMPLE EXAM 1

1. Let P and Q be statements. Fill out the truth table for $(P \implies Q) \wedge (Q \implies P)$. Is this a tautology, a contradiction, or neither?

2. Show that for any sets A, B, CD, we have
$$(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D).$$

3. Show that if p is a prime and a, b are natural numbers with $p|ab$, then $p|a$ or $p|b$.

4. Show that if n is a composite number, then there is a prime number p dividing n satisfying $p \leq \sqrt{n}$.

5. Show that $\sqrt[3]{2}$ is irrational.

6. Show that if x and y are real numbers with $0 < x < y$, then $\sqrt{x} < \sqrt{y}$. (Don't use calculus for this problem).