Math 2106-D, Foundations of Mathematical Proof
Homework 6
Due October 19, 2017

**Do the following problems from Hammack:**
Section 12.6: 10
Section 13.1: 6,8,10,14
Section 13.2: 4,6,8

**Also turn in the following exercise:**

A1 In class, we showed that if two sets $A$ and $B$ are countable (i.e., either finite or countably infinite), then $A \cup B$ is countable too. Using something known as the *axiom of choice*, one can show the following

**Theorem.** *A countable union of countable sets is countable.*

A real number is *algebraic* if it is the root of some polynomial with integer coefficients. For example, $\sqrt{2}$ is algebraic as it is a root of $x^2 - 2$, and the real root $x = -0.978322\dots$ of $7x^5 - 3x^4 + x + 10$ is algebraic.

Use this theorem to show that the set of algebraic numbers is countable. Conclude from this that there is at least one number which is not algebraic (such a number is called *transcendental*).