Math 2106-D, Foundations of Mathematical Proof
Homework 5
Due October 12, 2017

Do the following problems from Hammack:
Chapter 12.1: 4
Chapter 12.2: 10,12
Chapter 12.3: 2
Chapter 12.4: 8
Chapter 12.5: 4,8

Also turn in the following exercises:

A1 Consider the sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987\ldots$$

of Fibonacci numbers. For fixed $n \in \mathbb{N}$, we can look at the sequence of Fibonacci numbers modulo n by reducing each element of the sequence modulo n. For example, when $n = 3$, we get the sequence of numbers

$$0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0\ldots$$

Note that this sequence seems to be *repeating*. In particular, if you kept computing, you would find that this sequence is just the sequence $0, 1, 1, 2, 0, 2, 2, 1$ repeated over and over again.

Use the Pigeonhole Principle to show that this kind of pattern always occurs. That is, show that for any $n \in \mathbb{N}$, that the sequence of Fibonacci numbers modulo n is periodic.

A2 Find a bijection $f\colon \mathbb{N} \to \mathbb{Z}$ (hint: define a piecewise function which has different branches depending on whether $n \in \mathbb{N}$ is even or odd).