Math 2106-D, Foundations of Mathematical Proof
Homework 4
Due October 5, 2017

Do the following problems from Hammack:
  Chapter 10: 4,16,20,26
  Section 11.0: 10
  Section 11.1: 4,8
  Section 11.2: 12
  Section 11.3: 4
Also turn in the following exercises:

A1 Suppose that a football team can only score field goals (worth 3 points) and touchdowns (worth 7 points). Use induction to show that the football team can score any integral number of points $n \geq 12$ (of course, this isn't practical in a real game, but suppose that a game can go on indefinitely). (Hint: Show that it is possible to score 12, 13, or 14 points, and start the inductive hypothesis after this point.)

A2 Recall that the *n-th harmonic number* is the finite sum

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n-1} + \frac{1}{n} = \sum_{j=1}^{n} \frac{1}{j}.$$

Use induction to show that $H_{2^n} \geq 1 + n/2$. Conclude that the harmonic series $1 + 1/2 + 1/3 + \ldots$ diverges. (Hint: write down the first few cases of the inequality you need for the inductive step with explicit numbers).