

Math 4150-B, Foundations of Mathematical Proof  
SAMPLE FINAL EXAM

- Compute the Jacobi symbols  $\left(\frac{120}{91}\right)$  and  $\left(\frac{120}{93}\right)$ .
  - For which primes  $p$  is 10 a square modulo  $p$ ?
- Show that the Diophantine equation  $y^2 = x^3 - 5$  has no integral solutions.
- For each of the following equations, determine whether or not there are any integral solutions. If there are, find all solutions.
  - $13x + 30y = 19$
  - $8x - 12y + 20z = 6$
- Show that if the arithmetic function  $f(n)$  is multiplicative, then so is the function

$$g(n) = \sum_{d|n} f(d)^5.$$

- Prove that there are only finitely many rational numbers  $\frac{m}{n}$  such that

$$\left| \sqrt{2} - \frac{m}{n} \right| < \frac{1}{n^3}.$$

- Find a Pythagorean triple such that the difference of the first two (smaller) numbers is 1, and every entry is at least 100.
- Show that if  $E : y^2 = x^3 + Ax + B$  is an elliptic curve over the real numbers, and if  $P \neq 0$  (i.e.,  $P$  is not the point at infinity) is a point on  $E$  which satisfies  $2P = 0$ , then  $P = (x, 0)$  where  $x$  is a root of  $x^3 + Ax + B$ .