

Math 4150-B, Foundations of Mathematical Proof
SAMPLE EXAM 1

Throughout, x, a, b , and c denote integers.

1. (a) Use the Euclidean algorithm to find the gcd of 24 and 15 (you may not use prime factorizations in this question).
(b) Use your work from part (a) to express this gcd as an integer linear combination of 24 and 15.
2. (a) Show that c is a common multiple of a and b if and only if it is a multiple of the least common multiple of a and b .
(b) Show that if $c|ab$, then $c|(a, c)(b, c)$.
3. Show that there are infinitely many prime numbers congruent to 5 modulo 6.
4. Use Hensel's Lemma to find all integer solutions to the polynomial congruence
$$x^3 + 8x - 5 \equiv 0 \pmod{125}.$$
5. Find all solutions to the equation $x^2 \equiv 1 \pmod{441}$. (Hint: Use the Chinese Remainder Theorem to reduce this to the study of two equations modulo smaller numbers. It may also be useful to consider Hensel's Lemma.)