Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

GROUPS **Trinity Term 2016**

MA1132: Advanced Calculus SAMPLE EXAM

DAY **PLACE** **TIME**

Prof. Larry Rolen

Instructions to Candidates:

Attempt all questions. All questions will be weighted equally.

Materials Permitted for this Examination:

Formulae and Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used. This is a closed-book exam, so no notes or other study materials are allowed.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Suppose a curve is given by the parametric equations

$$\begin{cases} x = \frac{t^2}{2} \\ y = \frac{4t^{\frac{5}{2}}}{5} \\ z = \frac{2t^3}{3}, \end{cases}$$

where $t > 0$.

(a) Find the unit tangent vector $T(t)$ to the curve as a vector-valued function of t.

(b) Compute the unit normal vector $N(t)$.

(c) Compute the unit binormal vector $B(t)$.

(d) Compute the curvature as a function of t.

2. (a) Find the directional derivative of $f(x, y) = \cos(xy) - x$ at the point $(0, 1)$ in the direction of the vector $(1, 3)$.

(b) Consider the function

$$f(x, y, z) = \sqrt{\frac{z - xy}{x + z}}$$

and the point $P = (1, 2, 3)$. At the point P, in which direction does f increase the fastest? That is, find a vector pointing in the direction of largest increase. Also find the magnitude of this rate of increase in that direction.

(c) Use the chain rule to show that the gradient of a $\mathcal{C}^1$ (continuous first order partial derivatives) function $f(x, y)$ at any point $P \in \mathbb{R}^2$ is perpendicular to the level curve of f through P, if it isn't equal to 0.

3. (a) Consider the function $f(x, y) = x^2 - x + \cos(xy)$. Find all critical points of f, and decide which are local maxima, local minima, saddle points, or for which the second derivative test is inconclusive.

(b) The Extreme Value Theorem guarantees that the function $f(x, y) = x^3 + y^3$ has a global maximum value and a global minimum value on the circle $x^2 + y^2 = 1$. Use the method of Lagrange multipliers to find these values.

4. (a) Consider the cone given by the equation $z = \sqrt{x^2 + y^2}$ bounded above by the plane $z = 5$. Find parametric equations for this surface.

(b) Write the general equation for the surface area of a surface $x = x(u, v), y = y(u, v), z = z(u, v)$ determined by $(u, v) \in R$, where R is a region in the u-v plane.

(c) Use (a) and (b) to compute the surface area of this cone.