TUTORIAL 8

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^5) y dx dy$$

by switching the order of integration. That is, write this as a double integral over a region R in the plane (sketch a picture), rewrite this as in integral with respect to $dydx$, and evaluate.

(2) Evaluate

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{dydx}{\sqrt{x^2+y^2}}$$

by first switching to polar coordinates.

(3) Show that the parametric surface

$$\begin{cases} x = u\cos v \\ y = u\sin v \\ z = u^2 \end{cases}$$

with $1 \leq u \leq 4$ and $0 \leq v \leq \frac{\pi}{2}$ is a piece of the paraboloid $z = x^2 + y^2$. Find the surface area of this piece.

Advanced Problem:

The very important *Gamma function* is defined for positive real numbers as the integral

$$\int_0^\infty x^{z-1} e^{-x} dx.$$

Show that this function evaluates to $\Gamma(n) = (n-1)!$ at positive integers n. (Hint: use integration by parts to relate the values of $\Gamma(z)$ and $\Gamma(z+1)$.) The values at other places are interesting as well. Use a u-substitution to evaluate $\Gamma(1/2)$. Put together what you learned in the last two parts to find $\Gamma(n/2)$ for any positive integer n.