TUTORIAL 7

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Using the method of Lagrange multipliers, find the point on the plane $x-y+3z=1$ closest to the origin.

(2) Compute the double integral

$$\int_0^1 \int_0^{\sqrt{\log 2}} xye^{x^2} dxdy.$$

(3) Find the volume under the surface $z = \frac{x}{y}$ and above the rectangular region $R = [0,2] \times [1,3]$ in the x-y plane.

Advanced Problem: Suppose that $A = (A_{ij})$ is a symmetric, real-valued $n \times n$ matrix. Define a function $f \colon \mathbb{R}^n \to \mathbb{R}$ by the dot product $f(x) = x \cdot Ax$. Show that the largest and smallest values of f on the unit sphere $\{x \in \mathbb{R}^n | |x| = 1\}$ are the largest and smallest eigenvalues of A. (Hints: Use the method of Lagrange multipliers. What is ∇f? What is ∇g where $g(x) = x \cdot x - 1$? Try writing down a few examples for small n first.) Deduce that every real-valued, symmetric $n \times n$ matrix has at least one real eigenvalue.