HOMEWORK 9

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Find the surface area of the piece of the plane $z = x + y$ lying inside the cylinder $(x - 2)^2 + (y - 3)^2 = 1$.

(2) Gabriel's horn is a famous shape obtained by rotating the area under the curve $y = 1/x$ in the x-y plane from $x = 1$ to ∞ around the x-axis. Find parametric equations for this surface, and find an integral expression for the surface area of the "truncated" horn from $x = 1$ to $x = a$. Conclude, by using a comparison with a divergent integral, that this horn has infinite surface area.

(3) Evaluate the iterated integral

$$\int_{-1}^{1} \int_{-x}^{x} \int_{0}^{x^2+z} x \sin(x^7) y^2 \, dy \, dz \, dx.$$

(Hint: For the final integral over x, what do you notice about the integrand?)

(4) Find the volume of the region between the paraboloid $z = x^2 + y^2$ and the x-y plane above the annular region S lying between the concentric circles of radii 1 and 2 centered at the origin.