HOMEWORK 7

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Using the method of Lagrange multipliers, find the largest and smallest values of the function $f(x, y) = xy$ on the ellipse $\frac{x^2}{2} + \frac{y^2}{3} = 1$.

(2) Find the absolute maximum and minimum values of $f(x, y) = x^2 - y^2$ on the closed unit disc $\{(x, y) : x^2 + y^2 \leq 1\}$.

(3) As hinted at in class, there is a method of Lagrange multipliers not only for functions of many variables but for functions with multiple constraints. We will state the system of equations for a function subject to 2 constraints just for concreteness, but you should be able to guess the general formula from this example and the cases you have already seen! In this case, the system of equations to solve to search for extrema for a function $f(x_1, \ldots, x_n)$ subject to the constraints $g(x_1, \ldots, x_n) = 0$ and $h(x_1, \ldots, x_n) = 0$ is

$$\begin{cases} \nabla f = \lambda \nabla g + \mu \nabla h \\ g = 0 \\ h = 0. \end{cases}$$

Here, λ and μ are two real numbers. Note that there are $n + 2$ unknowns ($x_1, \ldots, x_n$ and λ, μ), and $n + 2$ equations in this system.

Use this method to find the closest point on the intersection of the surfaces $x^2 + y^2 = z^2$ and $x + y - z = 2$ to the origin.

(4) Evaluate the double integral

$$\int_2^3 \int_0^1 (4xy - x^2 + y) dy dx.$$

Also compute

$$\int_0^1 \int_2^3 (4xy - x^2 + y) dx dy$$

directly to see that you get the same answer.

(5) The average value of a two variable function $f(x, y)$ on a region R of the $x - y$ plane is the double integral of f over that region $\iint_R f(x, y) dA$, divided by the area of the region. Find the average value of $f(x, y) = x^2 + y^2 - 2xy$ on the box $[0, 1] \times [4, 5]$.