HOMEWORK 3

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) A particle moves along a curve in $\mathbb{R}^3$ with position function given by $r(t) = (e^t, \sqrt{t^2+1}, t)$. Find the velocity $v(t)$, the acceleration $a(t)$, the speed as a function of time, and the curvature κ. Further find the tangent and normal components of acceleration, a_T and a_N respectively, as functions of time.

(2) Suppose a particle travels through $\mathbb{R}^3$ with position vector given by $r(t) = \left(t\sqrt{2}, \frac{t^2}{2}, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\right)$. Find the distance the particle travels from $t = 0$ to $t = \sqrt{3} - 1$ (hint: use a trig integral and then integration by parts, and recall that $\int \sec\vartheta d\vartheta = \log|\sec\vartheta + \tan\vartheta| + C$).

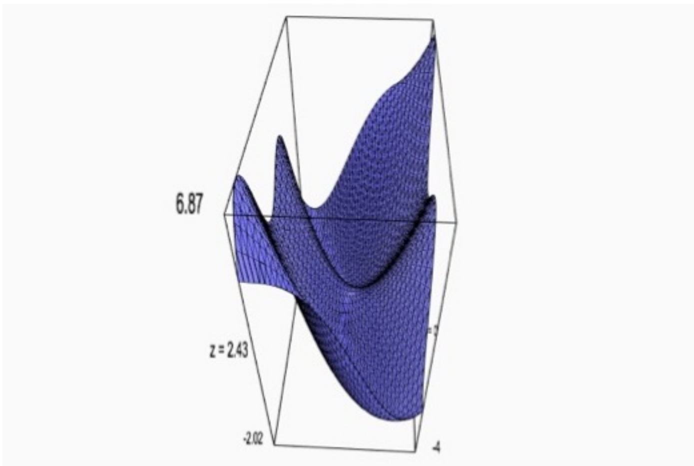
(3) You are standing on the edge of a Hag's Head, which is a cliff 120 m high above the Atlantic. Out of frustration with the lack of cell service, you throw your phone towards the ocean at an angle of 45° at a speed of 10 m/s. Find parametric equations describing the trajectory of the phone, and determine how far it travels horizontally before it hits the water.

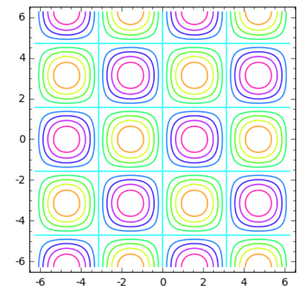
(4) Match the plots on the following page, which all illustrate the graph of some two-variable function, to their contour plots. The colors of the lines in the contour plots give an indication of the relative height of the graph on the level curves.

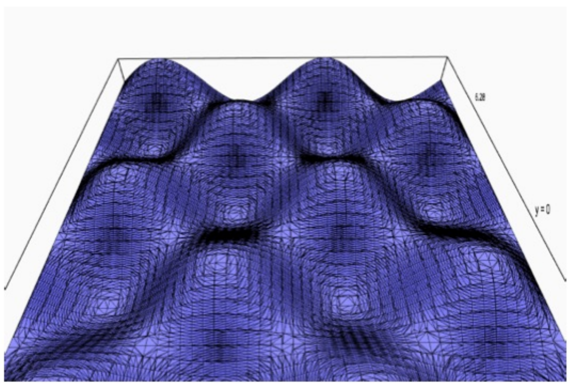
(5) Show that

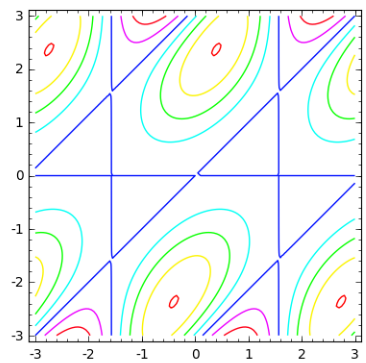
$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{(x+y)^2}$$

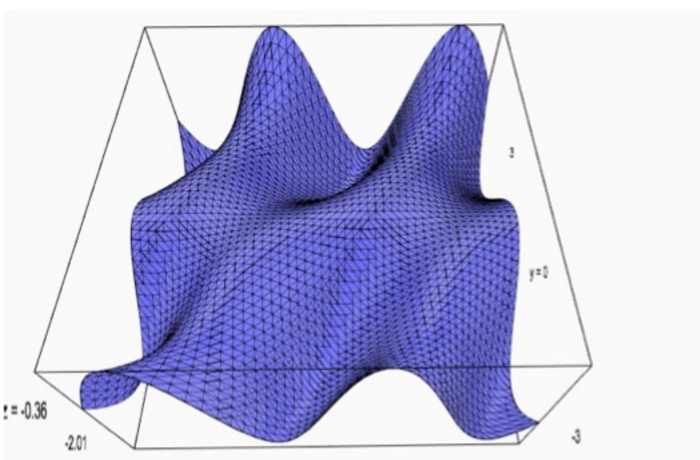
does not exist by finding two smoothly parameterized paths to the origin which give different limits as $(0,0)$ is approached.

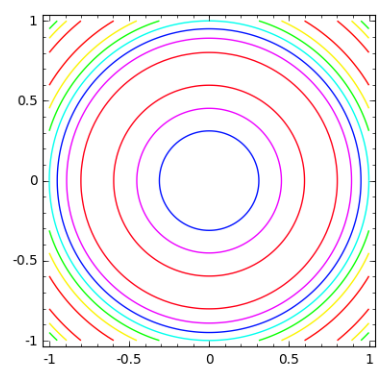

6.87
z = 2.43
-2.02
-4

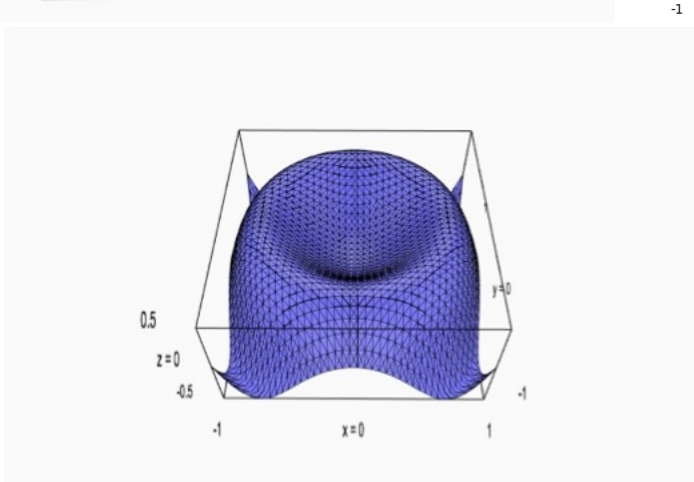

6
4
2
0
-2
-4
-6
-6 -4 -2 0 2 4 6

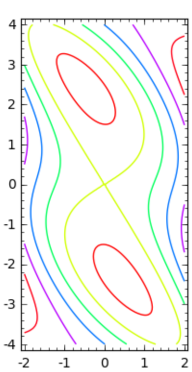

y = 0


3
2
1
0
-1
-2
-3
-3 -2 -1 0 1 2 3


3
y = 0
z = -0.36
-2.01
-3


1
0.5
0
-0.5
-1
-1 -0.5 0 0.5 1


y = 0
0.5
z = 0
-0.5
-1
-1
x = 0
1


4
3
2
1
0
-1
-2
-3
-4
-2 -1 0 1 2