HOMEWORK 2

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Find parametric equations for the tangent line of the graph of $r(t) = (t, \sqrt{t^2+1}, 3/t)$ when $t = 1$.

(2) The graphs of

$$r_1(t) = (t, 2t, t^2)$$

and

$$r_2(t) = (t^2, 2t^{\frac{3}{2}}, t^6)$$

intersect when $t = 1$ at the point $(1, 2, 1)$. Find the angle between the two tangent vectors of the graphs at this point.

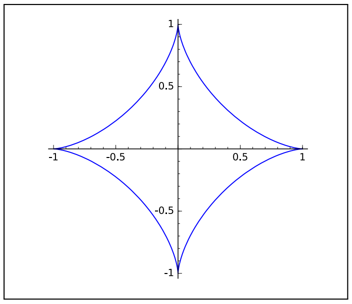
(3) (a) In single-variable calculus, you have been taught that the function $f(x) = |x|$ is not differentiable due to the sharp bend, or cusp, at the origin graph of $f(x)$. However, it is possible to find differentiable parameterizations of this curve, that is, parameterizations $r(t)$ for which $r'(t)$ exists for all t. Find one such parameterization.

(b) Just as graphs that as curves look to not be nice can have nice parameterizations, so can nice curves in the plane have not so nice parameterizations. Given any line parameterized by $r(t) = (x_0 + at, y_0 + bt, z_0 + ct)$ with $(a, b, c) \neq 0 \in \mathbb{R}^3$, note that $r'(t) = (a, b, c)$ is continuous and never vanishes, and hence that $r(t)$ is a smooth vector-valued function. Find a change of parameter $t = f(\tau)$ for which $r(f(\tau))$ parameterizes the same curve, but which does not do so smoothly.

(4) The curve parameterized by

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases}$$

for $0 \leq t < 2\pi$ is known as an *astroid*, and is pictured below. Find the arclength of this astroid (hint: it is useful to use the geometric symmetries of this figure; in particular, the length of the entire curve is the same as 4 times the length of the curve from $t = 0$ to $t = \pi/2$).



(5) Find the vector-valued functions $T(t), N(t)$, and $B(t)$ determining the unit tangent, unit normal, and binormal vectors to the helix with parameterization $r(t) = (\cos(t), \sin(t), t\sqrt{3})$.

(6) (a) Using your answer from 5), find the curvature of the helix $r(t) = (\cos(t), \sin(t), t\sqrt{3})$ as a function of t.

(b) The curvature of a curve measures how fast it turns, or, in a sense, how much it fails to be a straight line. There is another quantity, called *torsion*, which determines to what extent a space curve fails to lie in a plane. It is defined at any point on the curve by

$$\tau = -N \cdot \frac{dB}{ds}.$$

Compute the torsion of the helix from problem 5.