HOMEWORK 1 SOLUTIONS

MA1132: ADVANCED CALCULUS, HILARY 2017

(1) Plug in several values of t to guess what the sketch of the parametric curve given by
$$\begin{cases} x = t^2 - t \\ y = t - 1, \end{cases}$$
where $t \in \mathbb{R}$ looks like. Now sketch the parametric curve given by the same equations, but with the restriction on t given by $-1 \leq t \leq 1$. Clearly indicate the orientation of these graphs (the direction in which t is increasing).

(2) Prove that the sketch you drew in problem 1 matches your prediction by eliminating the parameter t (hint: solve for t in one of the parametric equations). That is, use algebra to rewrite the parametric equations in terms which allow you to immediately recognize what type of curve is given in problem 1.

(3) Find the domain of
$$\vec{r}(t) = \left(\log(t^2 - 1/4), \frac{1}{t+1}, \sqrt{2t+3} \right).$$

(4) Find a parametric representation of the curve of intersection of the cone $x^2+y^2 = z^2$, $z > 0$, and the plane $-x + z = 1$. Indicate in a sketch what this looks like, and describe this curve.

(5) Show that the graph of the vector-valued function
$$\vec{r}(t) = t\vec{i} + \frac{3-2t}{t}\vec{j} + \frac{(t-3)^2}{t}\vec{k}$$
($t \neq 0$) lies in the plane
$$x + 3y - z = 0.$$