Math 3100 Homework Assignment 7 - Due Friday April 9th

Your name here

April 1, 2021

Exercise 1. Suppose that $f : [a, b] \to \mathbb{R}$ is continuous and $\int_a^b |f| \, dx = 0$. Show that $f(x) = 0$ for all $x \in [a, b]$.

Exercise 2. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable at all points and there exits $M \geq 0$ such that $|f'(x)| \leq M$ for all $x \in \mathbb{R}$. Prove that f is uniformly continuous.

Exercise 3. Let $f : [0, 1] \to \mathbb{R}$ be monotone increasing. Prove that the sequence $\{a_n\}_{n=1}^{\infty}$ defined by $a_n = \frac{1}{n} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$ is convergent.

Exercise 4. Suppose $f : [a, b] \to \mathbb{R}$ is bounded and $\{c_n\}_{n=1}^{\infty}$ is a strictly increasing sequence in $[a, b]$ such that $c_1 = a$ and $\{c_n\}_{n=1}^{\infty}$ converges to b. Show if f is integrable on each interval $[c_i, c_{i+1}]$ for $i \geq 1$, then f is integrable on $[a, b]$.

Exercise 5. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and set $M = \sup\{|f(x)| \mid x \in [a, b]\}$. Prove that the sequence

$$\left\{ \left(\int_a^b |f(x)|^n \, dx \right)^{1/n} \right\}_{n=1}^{\infty}$$

converges to M.