Math 3100 Homework Assignment 6 - Due Wednesday March 17th

Your name here

March 15, 2021

Exercise 1. Suppose $D \subset \mathbb{R}$ and $x_0 \in D$ is an accumulation point of D. Suppose $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ are differentiable at x_0, and that $f(x) \leq g(x)$ for all $x \in D$, with $f(x_0) = g(x_0$. Prove that $f'(x_0) = g'(x_0)$.

Exercise 2. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable at all points and there exits $M \geq 0$ such that $|f'(x)| \leq M$ for all $x \in \mathbb{R}$. Prove that f is uniformly continuous.

Exercise 3. Use the Mean-Value theorem to prove that for $n \geq 1$ and $0 \leq x \leq y$ we have $ny^{n-1}(x - y) \leq x^n - y^n \leq nx^{n-1}(x - y)$.

Exercise 4. Suppose $f : [a, b] \to \mathbb{R}$ is twice differentiable and the second derivative satisfies $f''(x) \geq 0$ for all $x \in [a, b]$ prove that for $x, y \in [a, b]$ and $0 \leq t \leq 1$ we have $f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y)$.

Exercise 5. Suppose $f : [a, b] \to \mathbb{R}$ is differentiable and suppose f'' exists at some point $t \in (a, b)$. Prove that

$$f''(t) = \lim_{h \to 0} \frac{f(t + h) - 2f(t) + f(t - h)}{h^2}.$$

Give an example of some function f such that this limit exists for some t even though f' is not differentiable at t.