

Math 3100 Homework Assignment 5 - Due Friday
March 12th

Your name here

March 7, 2021

Exercise 1. Suppose $f : [a, b] \rightarrow [a, b]$ is continuous. Prove that there exists $c \in [a, b]$ such that $f(c) = c$.

Exercise 2. Prove that there does not exist a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that the equation $f(x) = c$ has exactly two solutions for each $c \in \mathbb{R}$.

Exercise 3. Find a function $f : [0, 1] \rightarrow \mathbb{R}$ that is not continuous, but still satisfies the intermediate value property, i.e., for each $0 \leq s < t \leq 1$ and $c \in [f(s), f(t)] \cup [f(t), f(s)]$, there exists a point $r \in [s, t]$ such that $f(r) = c$.

Exercise 4. Show that if a function $f : [0, 1] \rightarrow \mathbb{R}$ satisfies the intermediate value property, then either f is continuous or else there exists some $c \in \mathbb{R}$ such that the equation $f(x) = c$ has infinitely many solutions.