Math 3100 Homework Assignment 4 - Due Wednesday March 3rd

Your name here

February 26, 2021

Exercise 1. Let $f : \mathbb{R} \to \mathbb{R}$ be 1-1 and locally monotone in the sense that for every point $t_0 \in \mathbb{R}$ there exists $\varepsilon > 0$ such that the restriction of f to the interval $(t_0 - \varepsilon, t_0 + \varepsilon)$ is monotone. Prove that f is monotone on all of $\mathbb{R}$.

Exercise 2. A function $f : \mathbb{R} \to \mathbb{R}$ is periodic if there exists some $t_0 > 0$ such that $f(x + t_0) = f(x)$ for all $x \in \mathbb{R}$. Prove that if $f : \mathbb{R} \to \mathbb{R}$ is periodic and continuous then f is uniformly continuous.

Exercise 3. Suppose $A, B \subset \mathbb{R}$ are sets and $f : A \cup B \to \mathbb{R}$ is continuous such that $f_{|A}$ and $f_{|B}$ are each uniformly continuous. Must f be uniformly continuous? Either prove or give a counter-example. What if A and B are closed subsets?

Exercise 4. Let $\{G_\alpha\}_{\alpha \in A}$ be an arbitrary family of open subsets of $\mathbb{R}$. Prove that $\cup_{\alpha \in A} E_\alpha$ is open. Also, show that if $G_1, \ldots, G_n \subset \mathbb{R}$ are open sets then $\cap_{i=1}^{n} G_i$ is open.

Exercise 5. Let $A \subset \mathbb{R}$ be a bounded set and suppose $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are each uniformly continuous. Show that $fg : A \to \mathbb{R}$ given by $(fg)(t) = f(t)g(t)$ is also uniformly continuous.