Math 3100 Homework Assignment 3 - Due Wednesday February 17th

Your name here

February 11, 2021

Exercise 1. Suppose $0 \leq a \leq b$. Determine for what values of a and b the sequence $\{(a^n + b^n)^{1/n}\}_{n=1}^{\infty}$ has a limit. When it does, compute the limit and justify your answer.

Exercise 2. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers and let x be any accumulation point of the set of values $\{a_n \mid n \in \mathbb{Z}_+\}$. Prove that there is some subsequence that converges to x.

Exercise 3. Fix $x \in \mathbb{R}$. Prove that there exists a sequence $\{a_k\}_{k=1}^{\infty}$ taking values in the set $\{-1, 1\}$ such that the sequence $\{\sum_{k=1}^{n} \frac{a_k}{k}\}_{n=1}^{\infty}$ converges to x.

Exercise 4. Recall that any number $x \in [0, 1)$ has a unique representation in base-2 so that it does not end in a string of 1's, i.e., for each $x \in [0, 1)$ there is a unique sequence $\{a_n\}_{n=1}^{\infty}$ taking values in the set $\{0, 1\}$ such that the sequence is not eventually equal to 1 and such that x has the binary expansion

$$x = 0.a_1 a_2 a_3 \cdots ,$$

or equivalently $x = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{a_k}{2^k}$. Consider the function $f : [0, 1) \to \mathbb{R}$ that takes the binary representation of x and reinterprets it as a decimal representation, e.g., if x is as before then $f(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{a_k}{10^k}$. Determine all values in $[0, 1]$ where this function has a limit. Justify your answer.