# Math 3100 Homework Assignment 2 - Due Wednesday February 10th

Your name here

February 2, 2021

**Exercise 1.** For a sequence of real numbers $\{a_n\}_{n=1}^{\infty}$, and a real number $A \in \mathbb{R}$ consider the following conditions:

$$\forall N \geq 1, \exists \varepsilon > 0 \text{ such that } \forall n \geq N \text{ we have } |A - a_n| \leq \varepsilon. \tag{P1}$$

$$\exists N \geq 1 \text{ such that } \forall \varepsilon > 0, \text{ and } n \geq N \text{ we have } |A - a_n| \leq \varepsilon. \tag{P2}$$

$$\forall \varepsilon > 0, \exists N \geq 1 \text{ such that } \forall n \geq N \text{ we have } |A - a_n| \leq \varepsilon. \tag{P3}$$

$$\forall N \geq 1, \varepsilon > 0, \text{ and } n \geq N \text{ we have } |A - a_n| \leq \varepsilon. \tag{P4}$$

$$\forall \varepsilon > 0 \text{ and } N \geq 1, \exists n \geq N \text{ such that we have } |A - a_n| \leq \varepsilon. \tag{P5}$$

For each condition above give an intuitive description of the class of sequences that satisfy the condition and describe the relationship between those sequences and the number $A$.

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**Exercise 2.** If $S \subset \mathbb{R}$ is a set, let $L(S)$ denote the set of accumulation points of $S$. Prove that there either does or does not exist a set $S \subset \mathbb{R}$ such that $L(L(S)) \neq \emptyset$, but $L(L(L(S))) = \emptyset$.

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**Exercise 3.** Suppose $S \subset \mathbb{R}$ is an uncountable set. Prove that $S$ has an accumulation point.

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**Exercise 4.** Given a number $x \in \mathbb{R}$ we let $\lfloor x \rfloor$ denote floor function of $x$, i.e., $\lfloor x \rfloor$ is the greatest integer that is less than or equal to $x$. Prove that for all $x \in \mathbb{R}$ the sequence

$$\left\{ \frac{\lfloor x \rfloor + \lfloor 3x \rfloor + \cdots + \lfloor (2n+1)x \rfloor}{n^2} \right\}_{n=1}^{\infty}$$

converges to $x$.

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**Exercise 5.** Given a sequence $\{a_n\}_{n=1}^{\infty}$ we may define a new sequence $\{\alpha_n\}_{n=1}^{\infty}$ by $\alpha_n = \frac{a_1+a_2+\cdots+a_n}{n}$. Show that if $\{a_n\}_{n=1}^{\infty}$ converges to $A \in \mathbb{R}$ then we also have that $\{\alpha_n\}_{n=1}^{\infty}$ converges to $A$. Give an example where $\{a_n\}_{n=1}^{\infty}$ is divergent but $\{\alpha_n\}_{n=1}^{\infty}$ is convergent.

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**Exercise 6.** Prove that if $\{a_n\}_{n=1}^{\infty}$ converges to $A$, with $A \geq 0$ and $a_n \geq 0$ for all $n \geq 1$, then the sequence $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to $\sqrt{A}$.