

Math 3100 Homework Assignment 1 - Due Wednesday February 3rd

Your name here

January 27, 2021

Exercise 1. Give an uncountable family of infinite subsets of \mathbb{N} such that any distinct two of these sets intersect in a set with no more than 2021 elements, or else prove that no such family exists.

Exercise 2. Let A be a set. The **power set** of A is the set $\mathcal{P}(A)$ that contains all subsets of A . For example $\mathcal{P}(\emptyset) = \{\emptyset\}$, and $\mathcal{P}(\{0, 1\}) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$. Prove that for any set A there is no surjective function $f : A \rightarrow \mathcal{P}(A)$.

Exercise 3. A real number is called **algebraic** if it is a root of a polynomial having integer coefficients. For example any rational number $\frac{a}{b}$ is a root of the polynomial $bx - a$, or the number $\sqrt{2} + 1$ is a root of the polynomial $(x - 1)^2 - 2 = x^2 - 2x - 1$. Prove that there exist real numbers that are not algebraic.

Exercise 4. Prove that for every natural number $n \in \mathbb{N}$ the number $n^3 + 5n$ is divisible by 6.