MATH 6101 - HOMEWORK ASSIGNMENT 4

DUE THURSDAY, FEBRUARY 16TH BY 6:00PM

Exercise 0.1. Define $\varphi_n \in \ell^\infty(\mathbb{N})^*$ by $\varphi_n(f) = \frac{1}{n}\sum_{k=1}^n f(n)$, show that if φ is a weak* cluster point of $\{\varphi_n\}_n$ then $\varphi \notin \ell^1(\mathbb{N})$.

Exercise 0.2. Let X be a Banach space and suppose $\{x_n\}_{n=1}^\infty \subset X$ is a sequence which converges weakly, then $\{x_n\}_{n=1}^\infty$ is bounded.

Exercise 0.3. Suppose $x \in [0,1]$ has unique decimal expansion

$$x = 0.a_1a_2a_3\ldots.$$

Define

$$A_n(x) = \begin{cases} 1, & \text{if } a_n \text{ is even} \\ -1, & \text{if } a_n \text{ is odd.} \end{cases}$$

Note that A_n is defined almost everywhere. Show that the sequence A_n converge to 0 in the weak*-topology of $L^\infty([0,1],\lambda) = L^1([0,1],\lambda)^*$.

Exercise 0.4. Let (X,μ) be a σ-finite measure space. Then $L^1(X,\mu)$ is reflexive if and only if $L^1(X,\mu)$ is finite dimensional.

Exercise 0.5 (The Markov-Kakutani fixed point theorem)**.** Let $K \subset X$ be a non-empty compact convex subset of a locally convex space X, and suppose $\mathcal{S}$ is a non-empty family of pairwise commuting continuous maps which are affine, i.e., $T(tk_1 + (1-t)k_2) = tT(k_1) + (1-t)T(k_2)$ whenever $T \in \mathcal{S}$, $k_1, k_2 \in K$ and $t \in [0,1]$. Then there is a point in K which is a common fixed points for all maps in $\mathcal{S}$. Hint: First consider the case when $\mathcal{S}$ consists of a single map T, take $k_0 \in K$ and consider the sequence $\frac{1}{n}\sum_{i=0}^{n-1} T^i k_0$.

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