# MATH 6101 - HOMEWORK ASSIGNMENT 1

DUE THURSDAY, JANUARY 26TH BY 6:00PM

**Exercise 0.1** (The Lebesgue density topology on $\mathbb{R}$)**.** For a Lebesgue measurable set $A \subset \mathbb{R}$, set

$$D(A) = \left\{ x \in \mathbb{R} \mid \lim_{x \in I, \mathrm{diam}(I) \to 0} \frac{\lambda(A \cap I)}{\lambda(I)} = 1 \right\}.$$

We define a topology on $\mathbb{R}$ by letting a base for the topology be given by all measurable sets $A$ such that $A \subset D(A)$. Give a description of meager sets and use this to show that $\mathbb{R}$ with the Lebesgue density topology is a Baire space.

**Exercise 0.2.** There is a continuous function $f : \mathbb{R} \to \mathbb{R}$ which is not of bounded variation on any interval of positive length.

**Exercise 0.3.** There is a function of bounded variation $f : \mathbb{R} \to \mathbb{R}$ which is not monotone on any interval of positive length.

**Exercise 0.4.** The sum and product of two absolutely continuous function on a compact interval remains absolutely continuous.

**Exercise 0.5.** Let $\mu$ be a complex Borel measure on $\mathbb{R}$ and set $f(x) = \mu((-\infty, x])$. Then $f$ is absolutely continuous if and only if $\mu \ll \lambda$, and $f$ is singular if and only if $\mu \perp \lambda$.

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*Date*: January 25, 2017.