MATH 390 - VON NEUMANN ALGEBRAS - SPRING 2012

Instructor: Jesse Peterson
e-mail: jesse.d.peterson@vanderbilt.edu
Office: SC 1414
Office Hours:
Mondays: 10:10-11:00am
Wednesdays: 1:00-2:40pm
Or by appointment

Website: http://www.math.vanderbilt.edu/~peters10/teaching/spring2013/math390.html

Prerequisites: An introductory course in Functional Analysis.

Recommended books:
An Introduction to Operator Algebras by Kehe Zhu.
ISBN: 0849378753.

Jones' notes available at: http://math.berkeley.edu/~vfr/MATH20909/VonNeumann2009.pdf

Description:
In this course we will study von Neumann algebras with emphasis on II_1 factors. The course will start from the basics and end with recent research in the area. To start we will quickly go though some preliminary concepts often covered in an introductory course in Operator Algebras. These topics include:

- The Spectral Theorem and abelian von Neumann algebras
- Steinspring's Dilation Theorem
- Von Neumann's Double Commutant Theorem
- Kaplansky's Density Theorem
- Geometry of projections
- Classification into types

Next we will discuss some intermediate topics including:

- Constructing the trace for II_1 factors
- Group von Neumann algebras and the group-measure space construction
- Amenable von Neumann algebras
- Correspondences
- The basic construction
- Popa's Intertwining Theorem

To end we will discuss some advanced topics. These will be determined by class interest, and time left in the semester. The following are potential topics, the first five are related to Popa's Deformation/Rigidity Theory.

- Calculating fundamental groups and outer automorphism groups
- Solidity and primeness
- Strong solidity and and factors with unique Cartan subalgebra
- W^*-superrigidity
- Quantum Dirichlet forms
- Measured group theory
- Characters on countable groups
- Connes' theorem on uniqueness of separable amenable II_1 factors
- Selected topics from Voiculescu's Free Probability

- Type III factors
- Non-commutative Poisson boundaries

A course in Operator Algebras is not a prerequisite but a student not previously having such a course will need to put in some extra effort in the beginning. We will assume familiarity with some basic concepts in Operator Algebras such as continuous functional calculus, the Gelfand transform, and the GNS construction. See, for instance, the preliminary notes on the course website, or the first two chapters of Zhu's book.

Grades:

Grades will be based on class attendance/participation, homework (which will occasionally be assigned), and an in class presentation.

Seminar:

The Subfactor Seminar is held each Friday from 4:10-5:30pm in SC 1432. This seminar focuses on topics from von Neumann algebras, subfactors, and related areas. Students who are planning to work in the area of operator algebras are expected to regularly attend the seminar. It is also the best opportunity for students who are considering working in the area and would like to get a feel for the subject. After the students are welcome to join us for beer and pizza.

Summer schools:

There will be two summer school programs this summer in Europe related to von Neumann algebras. The first is a two month program, which includes a research conference, and starts in mid May:"Harmonic Analysis and Quantum Information Theory", at Instituto de Ciencias Mathemáticas, Spain. And the second is a two week program starting in mid June: "Rigidity and Group Actions", at Institut de Mathématiques de Jussieu, France. The second program has a follow up week long research conference: "Von Neumann Algebras and Measurable Group Theory", at Katholieke Universiteit Leuven, Belgium.

This course should provide all the needed foundations to attend these programs. Any students who would like to attend these programs should contact me in January so that we may begin to look for funding opportunities.