Math 208 - Quiz 9, April 2, 2009 Name:__________________Key______________________

Problem 1. Find the general solution to the differential equation $x^2y'' - 3xy' + 4y = 0$, for $x > 0$.

Solution 1. We first look for a solution of the form $y(x) = x^r$ for some $r \in \mathbb{R}$. In this case $y' = rx^{r-1}$ and $y'' = r(r-1)x^{r-2}$ hence $0 = x^2y'' - 3xy' + 4y = (r(r-1) - 3r + 4)x^r$.

Dividing by x^r (note $x \neq 0$) tells us that $0 = r^2 - 4r + 4 = (r-2)^2$, hence x^2 is a solution.

Since $r = 2$ is a repeated root we may find another solution by multiplying with $\ln x$. Since this is a 2nd order homogeneous linear equation the general solution is then given as

$$\boxed{y = c_1x^2 + c_2x^2 \ln x,}$$

where c_1 and c_2 are contants.