Math 208 - Quiz 8, March 26, 2009 Name:__________________Key______________________

Problem 1. Find a recurrence relation for the coefficients in a power series solution to differential equation $y'' + y' - xy = 0$ about the point $x_0 = 0$. Use this to calculate the first 4 coefficients to a solution which satisfies the initial value conditions $y(0) = 1$, $y'(0) = -1$.

Solution 1. Let $y = \Sigma_{n=0}^{\infty} a_n x_n$ be a power series solution to the above differential equation. Then

$$\begin{aligned}0 = y'' + y' - xy &= (\Sigma_{n=0}^{\infty} n(n-1)a_n x^{n-2}) + (\Sigma_{n=0}^{\infty} n a_n x^{n-1}) - x(\Sigma_{n=0}^{\infty} a_n x^n)\\ &= (\Sigma_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n) + (\Sigma_{n=0}^{\infty}(n+1)a_{n+1}x^n) - (\Sigma_{n=1}^{\infty} a_{n-1}x^n)\\ &= 2a_2 + a_1 + \Sigma_{n=1}^{\infty}((n+2)(n+1)a_{n+2} + (n+1)a_{n+1} - a_{n-1})x^n.\end{aligned}$$

Hence

$$\boxed{2a_2 + a_1 = 0}$$

and

$$\boxed{(n+2)(n+1)a_{n+2} + (n+1)a_{n+1} - a_{n-1} = 0}$$

whenever $n \geq 1$.

If $y(0) = 1$ and $y'(0) = -1$ then

$$\boxed{a_0 = 1, \quad a_1 = -1, \quad a_2 = -\frac{1}{2}a_1 = \frac{1}{2}, \quad a_3 = \frac{1}{6}(-2a_2 + a_0) = 0.}$$